Econ 203B: Single Equation Models

Solutions for 2001 Midterm

Michael Powell
Department of Economics, UCLA
February 3rd, 2006

1. (15 points) One aspect of the rational expectations hypothesis involves the claim that expectations are unbiased, that is, that the average prediction is equal to the observed realization of the variable under investigation. This claim can be tested by reference to announced predictions and actual values of the rate of interest on 3-month U.S. treasury Bills published in the Goldsmith-Nagan Bond and Money Market Letter. The results of Least Squares estimation (based on 30 quarterly observations) of the regression of the actual on the predicted interest rates were as follows:

$$r_t = 0.24 + 0.94 r_t^* + \hat{\varepsilon}_t$$

where r_t is the observed interest rate and r_t^* is the average expectation of r_t held at the end of the preceeding quarter. The sample data give

$$\sum_t r_t^* = 300 \quad \sum_t (r_t^* - \bar{r}^*)^2 = 52 \quad RSS = 28.56$$

where RSS stands for Residual Sum of Squares. Carry out a test (at the 5% significance level) of the rational expectations hypothesis using the results above, assuming that all basic assumptions of the classical normal regression model hold.

Solution The rational expectations assumption for the model:

$$r_t = \beta_1 + \beta_2 r_t^* + \varepsilon_t$$

is $H_0 : \beta_1 = 0,\ \beta_2 = 1$. Or, in matrix form, $H_0 : \Gamma\beta = \gamma_0$, where

$$\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \gamma_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

To test this, we would use the F statistic:

$$\begin{aligned} F_0 &= \frac{1}{p}\left(\Gamma\hat{\beta} - \gamma_0\right)'\left(\hat{\sigma}^2 \Gamma \left(X'X\right)^{-1}\Gamma'\right)^{-1}\left(\Gamma\hat{\beta} - \gamma_0\right) \\ &= \frac{1}{p}\frac{1}{\hat{\sigma}^2}\left(\hat{\beta} - \gamma_0\right)'\left(X'X\right)\left(\hat{\beta} - \gamma_0\right) \end{aligned}$$

It remains to calculate $\hat{\sigma}^2$ and $(X'X)$. Proceeding first with $\hat{\sigma}^2$:

$$\hat{\sigma}^2 = \frac{1}{n-k}\sum_{t=1}^{n}\hat{\varepsilon}_t^2 = \frac{1}{30-2}RSS = \frac{28.56}{28} = 1.02$$

Next, we have:

$$\begin{aligned} (X'X) &= \left(\begin{bmatrix} 1 & \cdots & 1 \\ r_1^* & \cdots & r_n^* \end{bmatrix}\begin{bmatrix} 1 & r_1^* \\ \vdots & \vdots \\ 1 & r_n^* \end{bmatrix}\right) \\ &= \begin{pmatrix} n & \sum_{t=1}^n r_t^* \\ \sum_{t=1}^n r_t^* & \sum_{t=1}^n (r_t^*)^2 \end{pmatrix} \end{aligned}$$

What is $\sum_{t=1}^{n} (r_t^*)^2$?

$$\begin{aligned}\sum_{t=1}^{30} (r_t^* - \bar{r}^*)^2 &= \sum_{t=1}^{30} \left[(r_t^*)^2 - 2\bar{r}^* r_t^* + (\bar{r}^*)^2\right] \\ &= \sum_{t=1}^{30} (r_t^*)^2 - 2\,(30)\,(\bar{r}^*)^2 + 30\,(\bar{r}^*)^2 \\ &= \sum_{t=1}^{30} (r_t^*)^2 - 30\,(\bar{r}^*)^2\end{aligned}$$

Rearranging, and recognizing that $\bar{r}^* = \frac{1}{30}\sum_t r_t^* = \frac{300}{30} = 10$, we have:

$$\begin{aligned}52 &= \sum_{t=1}^{30} (r_t^*)^2 - 3000 \\ \sum_{t=1}^{30} (r_t^*)^2 &= 3052\end{aligned}$$

This gives us:

$$(X'X) = \begin{bmatrix} 30 & 300 \\ 300 & 3052 \end{bmatrix}$$

And therefore,

$$\begin{aligned}F_0 = \frac{1}{2} \cdot \frac{1}{1.02} \left(\begin{bmatrix} 0.24 \\ 0.94 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)' \begin{bmatrix} 30 & 300 \\ 300 & 3052 \end{bmatrix} \left(\begin{bmatrix} 0.24 \\ 0.94 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\ = 1.9976\end{aligned}$$

The 5% critical value for an $F(2,28)$ distribution is $c^*_{0.05,F(2,28)} = 3.3404$. Since $F_0 \leq c^*_{0.05,F(2,28)}$, we fail to reject the rational expectations hypothesis.

2. (30 points) For each one of the following claims show whether they are true or false.

a. The R^2 of a k-variate regression does not change if we add to the dependent variable a constant and/or if we multiply the dependent variable by a constant.

Solution This question has two parts. First: Compare the R^2 from the following two regressions:

$$\begin{aligned}Y &= X\beta + \varepsilon \\ Y + c &\equiv \tilde{Y} = X\tilde{\beta} + \tilde{\varepsilon}\end{aligned}$$

The R^2 for the original model is:

$$R^2 = 1 - \frac{Y'M_X Y}{Y'M^0 Y}$$

The R^2 for the model with the constant is:

$$\tilde{R}^2 = 1 - \frac{\tilde{Y}'M_X \tilde{Y}}{\tilde{Y}'M^0 \tilde{Y}}$$

Expanding the numerator, we see:

$$\begin{aligned}\tilde{Y}'M_X\tilde{Y} &= (Y + \mathbf{c})'\,M_X\,(Y + \mathbf{c}) \\ &= Y'M_X Y + \mathbf{c}'M_X Y + Y'M_X\mathbf{c} + \mathbf{c}'M_X\mathbf{c}\end{aligned}$$

Recall that if X contains a constant column, then

$$\left[\begin{array}{cc} 0 & 0 \end{array}\right] = M_X X = M_X' X = M_X \left[\begin{array}{cc} i & X_{-1} \end{array}\right] = \left[\begin{array}{cc} M_X' i & M_x' X_{-1} \end{array}\right]$$

In particular, $M_X' i = 0$. Multiplying both sides by c, we have $M_X' \mathbf{c} = 0$ and, of course, $\mathbf{c}' M_X = (M_X' \mathbf{c})' = 0' = 0$. This gives us:

$$\tilde{Y}' M_X \tilde{Y} = Y' M_X Y$$

Similarly, expanding the numerator, we see:

$$\begin{aligned} \tilde{Y}' M^0 \tilde{Y} &= (Y + \mathbf{c})' M^0 (Y + \mathbf{c}) \\ &= Y' M^0 Y + \mathbf{c}' M^0 Y + Y' M^0 \mathbf{c} + \mathbf{c}' M^0 \mathbf{c} \end{aligned}$$

Since $M^0 = \left(I - i\,(i'i)^{-1}\,i'\right)$, clearly, $i' M^0 = i' - i'i\,(i'i)^{-1}\,i' = i' - i' = 0$ and thus, if we multiply both sides by c, we have $\mathbf{c}' M^0 = 0$. Therefore,

$$\tilde{Y}' M^0 \tilde{Y} = Y' M^0 Y$$

Putting this together, I have shown that if the original regression has a constant term, then

$$\tilde{R}^2 = 1 - \frac{\tilde{Y}' M_X \tilde{Y}}{\tilde{Y}' M^0 \tilde{Y}} = 1 - \frac{Y' M_X Y}{Y' M^0 Y} = R^2$$

The second part of the question asks us to compare the R^2 values from:

$$\begin{aligned} Y &= X\beta + \varepsilon \\ cY &\equiv \tilde{Y} = X\tilde{\beta} + \tilde{\varepsilon} \end{aligned}$$

Proceeding as in the previous part,

$$\tilde{R}^2 = 1 - \frac{\tilde{Y}' M_X \tilde{Y}}{\tilde{Y}' M^0 \tilde{Y}}$$

Expanding the numerator, we have:

$$\tilde{Y}' M_X \tilde{Y} = cY' M_X cY = c^2 Y' M_X Y$$

Expanding the denominator,

$$\tilde{Y}' M^0 \tilde{Y} = cY' M^0 cY = c^2 Y' M^0 Y$$

This gives us:

$$\tilde{R}^2 = 1 - \frac{c^2 Y' M_X Y}{c^2 Y' M^0 Y} = 1 - \frac{Y' M_X Y}{Y' M^0 Y} = R^2$$

b. The residuals from the regression of Y on X_1 and X_2 coincide with the residuals from the "residual" regression of Y on $\tilde{X}_1$ where $\tilde{X}_1$ are the residuals from the regression of X_1 on X_2.

Solution

Here, we are asked to compare the residuals from the following two regressions:

$$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon \tag{1}$$
$$Y = \tilde{X}_1\tilde{\beta}_1 + \tilde{\varepsilon} \tag{2}$$

where

$$\tilde{X}_1 = M_{X_2} X_1$$

For what follows, define $\tilde{X}_2 = M_{X_1} X_2$. If we estimate (1) using the partitioned regression formula, we have:

$$\begin{aligned}
\hat{Y}^1 &= X_1 \hat{\beta}_1 + X_2 \hat{\beta}_2 \\
&= X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y + X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' Y \\
&= \left(X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' + X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' \right) Y
\end{aligned}$$

Which gives us:

$$\hat{\varepsilon}^1 = Y - \hat{Y}^1 = Y - X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y - X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' Y$$

Estimating (2) using standard OLS, we get:

$$\begin{aligned}
\hat{Y}^2 &= \tilde{X}_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y \\
&= M_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y
\end{aligned}$$

Which gives us:

$$\hat{\varepsilon}^2 = Y - \hat{Y}^2 = Y - M_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y$$

Putting this together, we have:

$$\hat{\varepsilon}^2 - \hat{\varepsilon}^1 = X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y + X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' Y - M_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y$$

A necessary condition for $\hat{\varepsilon}^1 = \hat{\varepsilon}^2$ is that for any conformable matrix A, $A\hat{\varepsilon}^1 = A\hat{\varepsilon}^2$. Therefore, if we can find some A such that $A\hat{\varepsilon}^1 \neq A\hat{\varepsilon}^2$, then we can conclude that $\hat{\varepsilon}^1 \neq \hat{\varepsilon}^2$. Let $A = P_{X_2}$. Then we have:

$$\begin{aligned}
P_{X_2} \hat{\varepsilon}^2 - P_{X_2} \hat{\varepsilon}^1 &= P_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y + P_{X_2} X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' Y - P_{X_2} M_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y \\
&= P_{X_2} X_1 \left(\tilde{X}_1' \tilde{X}_1 \right)^{-1} \tilde{X}_1' Y + X_2 \left(\tilde{X}_2' \tilde{X}_2 \right)^{-1} \tilde{X}_2' Y
\end{aligned}$$

Here, I used the facts that

$$P_{X_2} M_{X_2} = P_{X_2} \left(I - P_{X_2} \right) = P_{X_2} - P_{X_2} P_{X_2} = P_{X_2} - P_{X_2} = 0$$

And

$$P_{X_2} X_2 = X_2 \left(X_2' X_2 \right)^{-1} X_2' X_2 = X_2$$

Therefore, unless we simultaneously have $\tilde{X}_2' Y = 0$ (equivalently $\beta_2 = 0$) and either $P_{X_2} X_1 = 0$ (X_1 and X_2 are orthogonal) or $\tilde{X}_1' Y = 0$ (equivalently $\beta_1 = 0$) then it must be the case that

$$P_{X_2} \hat{\varepsilon}^1 \neq P_{X_2} \hat{\varepsilon}^2$$

Therefore, except in this weird (and probably impossible) case, we have found a conformable matrix A such that

$$A\hat{\varepsilon}^1 \neq A\hat{\varepsilon}^2$$

And can conclude that $\hat{\varepsilon}^1 \neq \hat{\varepsilon}^2$.

c. For the k-variate regression model, $y = X\beta + \varepsilon$, the fit as measured by R^2 does not change if we transform the X matrix by postmultiplying it by a $k \times k$ non-singular matrix.

Solution This question asks us to compare the R^2 values for the following two regressions:

$$\begin{aligned} Y &= X\beta + \varepsilon \\ Y &= XC\tilde{\beta} + \tilde{\varepsilon} \equiv \tilde{X}\tilde{\beta} + \tilde{\varepsilon} \end{aligned}$$

I will proceed by computing $M_{\tilde{X}}$:

$$\begin{aligned} M_{\tilde{X}} &= I - \tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}' \\ &= I - XC\left(C'X'XC\right)^{-1}C'X' \\ &= I - XCC^{-1}\left(X'X\right)^{-1}\left(C'\right)^{-1}C'X' \\ &= I - X\left(X'X\right)^{-1}X' = M_X \end{aligned}$$

Therefore, we have:

$$\tilde{R}^2 = 1 - \frac{Y'M_{\tilde{X}}Y}{Y'M^0Y} = 1 - \frac{Y'M_XY}{Y'M^0Y} = R^2$$

3. (10 points) Suppose you are a firm and are trying to decide how much research and development to undertake. The profit function for firms in your industry takes the form:

$$\Pi_i = \beta_1 + \beta_2 RD_i + \beta_3 RD_i^2 + u_i$$

where Π_i and RD_i are profits and research and development (R&D) expenditures for firm i, and where $\beta_2 > 0$ and $\beta_3 < 0$. Given data on profits and R&D expenditures for firms in your industry explain how you would use them to choose the optimal level of R&D.

Solution Taking expectations of both sides, we have:

$$E\left[\Pi_i | RD\right] = \beta_1 + \beta_2 RD_i + \beta_3 RD_i^2$$

From here, I would take first order conditions with respect to RD_i:

$$\frac{\partial E\left[\Pi_i | RD\right]}{\partial RD_i} = \beta_2 + 2\beta_3 RD_i^* = 0$$

$$RD_i^* = -\frac{\beta_2}{2\beta_3}$$

This is necessarily a maximum, since, given that $\beta_2 > 0$, and $\beta_3 < 0$, we have that the second order conditions are:

$$\frac{\partial^2 E\left[\Pi_i | RD\right]}{\partial RD_i^2} = 2\beta_3 < 0$$

Under the CLR assumptions, $\hat{\beta}$ is the BLUE for β. Therefore, I would use the data given to estimate the model:

$$\hat{\Pi}_i = \hat{\beta}_1 + \hat{\beta}_2 RD_i + \hat{\beta}_3 RD_i^2$$

And choose

$$RD_i^* = -\frac{\hat{\beta}_2}{2\hat{\beta}_3}$$

4. (15 points) Consider the following regression model:

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + v_i$$

Explain in as much detail as you can how you would test for the following hypotheses:

(i) $\beta_2 = \beta_3$

(ii) $\beta_3 = 3\beta_4$

(iii) $\beta_2 = \beta_3 = \beta_4 = 0$

Solution Assuming all the assumptions of the CLR model hold, I would estimate this model:

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{i2} + \hat{\beta}_3 X_{i3} + \hat{\beta}_4 X_{i4}$$

Further, I would estimate the variance-covariance matrix $\hat{\sigma}^2 \left(X'X\right)^{-1}$ where

$$\hat{\sigma}^2 = \frac{1}{n-4}\left(Y - \hat{Y}\right)'\left(Y - \hat{Y}\right)$$

For (i), the null hypothesis can be written in matrix notation as:

$$H_0 : \Gamma\beta = \gamma_0$$

Where $\Gamma = \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}$ and $\gamma_0 = 0$. I would then proceed to calculate the test statistic:

$$F_0 = \left(\Gamma\beta - \gamma_0\right)'\left(\hat{\sigma}^2\Gamma\left(X'X\right)^{-1}\Gamma'\right)^{-1}\left(\Gamma\beta - \gamma_0\right)$$

And, assuming we wanted a $(1-\alpha)\,100\%$ significance level, I would compare F_0 to the critical value $c^*_{\alpha,F(1,n-4)}$, of course rejecting H_0 if $F_0 > c^*$ and failing to reject if $F_0 \leq c^*$

For (ii), the null hypothesis can be written in matrix notation as:

$$H_0 : \Gamma\beta = \gamma_0$$

Where $\Gamma = \begin{bmatrix} 0 & 0 & 1 & -3 \end{bmatrix}$ and $\gamma_0 = 0$. As above, I would calculate the test statistic:

$$F_0 = \left(\Gamma\beta - \gamma_0\right)'\left(\hat{\sigma}^2\Gamma\left(X'X\right)^{-1}\Gamma'\right)^{-1}\left(\Gamma\beta - \gamma_0\right)$$

Which I would compare to $c^*_{\alpha,F(1,n-4)}$. (Rejecting H_0 if $F_0 > c^*$ and failing to reject of $F_0 \leq c^*$)

Finally, for (iii), the null hypothesis can be written in matrix notation as:

$$H_0 : \Gamma\beta = \gamma_0$$

Where $\Gamma = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\gamma_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Once again, I would calculate the test statistic:

$$F_0 = \frac{1}{3}\left(\Gamma\beta - \gamma_0\right)'\left(\hat{\sigma}^2\Gamma\left(X'X\right)^{-1}\Gamma'\right)^{-1}\left(\Gamma\beta - \gamma_0\right)$$

Which I would compare to $c^*_{\alpha,F(3,n-4)}$. (Rejecting H_0 if $F_0 > c^*$ and failing to reject of $F_0 \leq c^*$)

5. (30 points) Suppose you have annual data from 50 different states over the last 20 years and you are trying to estimate the effect of different policies on the local unemployment rate. You run two regressions and obtain the following output:

Variable	Coefficient	Standard Error
Constant	0.013	0.05
Average Wage	-0.025	0.08
Minimum Wage	0.02	0.04
Average Welfare Benefits	0.031	0.012

Observations	1000
R^2	0.114

And

Variable	Coefficient	Standard Error
Constant	0.013	0.05
Average Wage	−0.035	0.06

Observations	1000
R^2	0.103

Carry the following tests at the 10% significance level assuming that the assumptions of the classical normal regression model hold.

(i) Test whether the value of minimum wage influences unemployment.

Solution Here, we want to test $H_0 : \beta_{\text{min wage}} = 0$ against $H_A : \beta_{\text{min wage}} \neq 0$. We use the test statistic $|t_0| = \left|\frac{\hat{\beta}_{\text{min wage}}}{se(\hat{\beta}_{\text{min wage}})}\right| = \left|\frac{0.02}{0.04}\right| = \frac{1}{2}$ and compare it to $c^*_{0.05,N(0,1)} = 1.645$. Clearly, $|t_0| \leq 1.645$, so we fail to reject the null hypothesis that minimum wage infludences unemployment.

(ii) Test whether welfare benefits increases unemployment.

Solution Here, we want to test $H_0 : \beta_{\text{welfare}} = 0$ against $H_A : \beta_{\text{welfare}} > 0$. Using the test statistic $t_0 = \frac{\hat{\beta}_{\text{welfare}}}{se(\hat{\beta}_{\text{welfare}})} = \frac{0.031}{0.012} = 2.5833$, we compare it to $c^*_{0.10,N(0,1)} = 1.28$. Clearly, $t_0 > c^*$ and we therefore reject the null in favor of the claim that welfare benefits increase unemployment.

(iii) Test jointly whether the minimum wage and welfare benefits influence unemployment.

Solution Here, we want to test $H_0 : \beta_{\text{welfare}} = \beta_{\text{min wage}} = 0$. Using the F statistic

$$F_0 = \left(\frac{n-k}{p}\right)\left(\frac{R^2_{UR} - R^2_R}{1 - R^2_{UR}}\right) = \left(\frac{1000-4}{2}\right)\left(\frac{.114 - .103}{1 - .114}\right) = 6.1828$$

Comparing this to the critical value: $c^*_{0.10,F(2,\infty)} = 9.49$, we see that $F_0 \leq c^*$ and we therefore fail to reject the null hypothesis that minimum wage and welfare benefits do not have any effect on unemployment.

(iv) Describe how you would test whether the effect of welfare benefits on unemployment is constant over time.

Solution I would estimate the following model:

$$unemp_i = \beta_1 + \beta_2 avgwage_i + \beta_3 minwage_i + \beta_4 welfare_i + \beta_5 t_i * welfare_i$$

Where t_i denotes the year of observation i. Performing a $t - test$ on the null hypothesis $H_0 : \beta_5 = 0$ using the statistic $|t_0| = \left|\frac{\hat{\beta}_5}{se(\hat{\beta}_5)}\right|$ and comparing it to the relevant critical value (here, 1.645), we would conclude that if $|t_0| \leq 1.645$, then the effect of welfare benefits on unemployment is constant over time. Otherwise, we reject the null.

(v) Describe how you would test whether the effect of minimum wage is constant over states.

Solution Here, I would estimate the model:

$$unemp_i = \beta_1 + \beta_2 avgwage_i + \beta_4 welfare_i + \sum_{s \in S} \beta_5^s state_i^s * minwage_i + \varepsilon_i \qquad (1)$$

Where $S = \{\text{Alabama}, \ldots, \text{Wyoming}\}$. Where $state_i^s$ is a dummy variable which takes on a value 1 if observation i is taken in state $s \in S$. We want to test the null hypothesis $H_0 : \beta_5^s = \beta_5^{s'}\ \forall s \neq s'$. In order to do so, I would run the unrestricted regression (1) above and acquire the unrestricted residuals $\hat{\varepsilon}_{UR}$. Then, I would run the restricted regression:

$$unemp_i = \beta_1 + \beta_2 avgwage_i + \beta_3 \text{minwage}_i + \beta_4 welfare + u_i$$

And acquire the restricted residuals $\hat{\varepsilon}_R$. Then, since this amounts to imposing 49 restrictions, we have that $p = 49$. Also, we know that $n = 1000$. What is k? Here, we are estimating 50 β_5^s parameters, a β_1, a β_2, and a β_4. That is, $k = 53$. Therefore, I would construct the following test statistic:

$$\begin{aligned} F_0 &= \frac{n-k}{p} \frac{\hat{\varepsilon}_R'\hat{\varepsilon}_R - \hat{\varepsilon}_{UR}'\hat{\varepsilon}_{UR}}{\hat{\varepsilon}_{UR}'\hat{\varepsilon}_{UR}} \\ &= \frac{947}{49} \frac{\hat{\varepsilon}_R'\hat{\varepsilon}_R - \hat{\varepsilon}_{UR}'\hat{\varepsilon}_{UR}}{\hat{\varepsilon}_{UR}'\hat{\varepsilon}_{UR}} \end{aligned}$$

And compare it to the critical value $c^*_{0.05,F(49,947)} = 1.37$. If $F_0 \leq c^*_{0.05,F(49,947)}$, I would not reject the null hypothesis that there are no state effects. Otherwise, I would conclude that the effect of minimm wage on unemployment is not constant over states.

(vi) Describe how you would test for the hypothesis whether the effect of minimum wage depends on average welfare benefits.

Solution I would estimate the model:

$$unemp_i = \beta_1 + \beta_2 avgwage_i + \beta_3 minwage_i + \beta_4 welfare_i + \beta_5 welfare_i * minwage_i$$

Performing a $t - test$ on the null hypothesis $H_0 : \beta_5 = 0$ using the statistic $|t_0| = \left|\frac{\hat{\beta}_5}{se(\hat{\beta}_5)}\right|$ and comparing it to the relevant critical value (here, 1.645), we would conclude that if $|t_0| > 1.645$, then the effect of minimum wage on unemployment depends on average welfare benefits.