Extra Office Hour: Wednesday, 12^{00} - 1^{30} Bunche 2265

MT1999

Question 3:

a] Will be discussed later

b] Original equation: $\hat{Q} = \hat{\beta}_1 + \hat{\beta}_2 P + \hat{\beta}_3 Y + \hat{\gamma}_1 S_1 + \hat{\gamma}_2 S_2 + \hat{\gamma}_3 S_3$

· What if we want to use S_2, S_3, and S_4 instead?

◦ $S_1 = 1 - S_2 - S_3 - S_4$

$$\Rightarrow \hat{Q} = \hat{\beta}_1 + \hat{\beta}_2 P + \hat{\beta}_3 Y + \hat{\gamma}_1 (1 - S_2 - S_3 - S_4) + \hat{\gamma}_2 S_2 + \hat{\gamma}_3 S_3$$
$$= \hat{\beta}_1 + \hat{\beta}_2 P + \hat{\beta}_3 Y + \hat{\gamma}_1 + (\hat{\gamma}_2 - \hat{\gamma}_1) S_2 + (\hat{\gamma}_3 - \hat{\gamma}_1) S_3 - \hat{\gamma}_1 S_4$$

Since $\hat{\beta}_1 = 70$, $\hat{\beta}_2 = -0.01$, $\hat{\beta}_3 = 0.2$, $\hat{\gamma}_1 = -1.5$, $\hat{\gamma}_2 = 3.6$, $\hat{\gamma}_3 = 4.7$,

$$\Rightarrow \hat{Q} = 68.5 - 0.01 P + 0.2 Y + 5.1 S_2 + 6.2 S_3 + 1.5 S_4$$

c] Someone wants to throw in all the dummies.

$1 = S_1 + S_2 + S_3 + S_4$

$$\Rightarrow \hat{Q} = \hat{\beta}_1 (S_1 + S_2 + S_3 + S_4) + \hat{\beta}_2 P + \hat{\beta}_3 Y + \hat{\gamma}_1 S_1 + \hat{\gamma}_2 S_2 + \hat{\gamma}_3 S_3$$
$$= \hat{\beta}_2 P + \hat{\beta}_3 Y + (\hat{\gamma}_1 + \hat{\beta}_1) S_1 + (\hat{\gamma}_2 + \hat{\beta}_1) S_2 + (\hat{\gamma}_3 + \hat{\beta}_1) S_3 + \hat{\beta}_1 S_4$$

Plugging in the estimated values,

$$\Rightarrow \hat{Q} = -0.01 P + 0.2 Y + 68.5 S_1 + 73.6 S_2 + 74.7 S_3 + 70 S_4$$

MT2000

Question 1: $\hat{Y} = 2.20 + 0.104 X_2 - 3.48 X_3 + 0.34 X_4$ (1)

MSS = 109.6 model sum of squares

ESS = 18.48 error sum of squares

With three seasonal dummies,

MSS' = 114.8

a] Test for seasonality

(1) is restricted model

Want to use $F_0 = \left(\frac{n-k}{p}\right)\left(\frac{ESS_R - ESS_U}{ESS_U}\right)$

- $TSS_R = MSS_R + ESS_R = 109.6 + 18.48 = 128.08$
- $ESS_{UR} = \underbrace{TSS_{UR}}_{=TSS_R} - MSS_{UR}$

$= 128.08 - 114.8 = 13.28$

$\Rightarrow F_0 = \left(\frac{76-7}{3}\right)\left(\frac{18.48 - 13.28}{13.28}\right) = (23)(0.3916) = 9.0060$

This is the test statistic for H_0: seasonal dummies have no effect

$c^*_{0.05, F(3,69)} = 2.7375$

Since $F_0 = 9.0060 > 2.7375 = c^*_{0.05, F(3,69)}$, we reject the null in favor of seasonality.

b) Do a Chow Test for structural change (see lecture notes 4, pp 18-19)

Split up the dataset: I: 58-I to 68-IV 44 observations
II: 69-I to 76-IV 32 observations

In period I: $ESS_I = 9.32$
II: $ESS_{II} = 7.46$

(1) $y_I = X_I \beta_I + \varepsilon_I$ Is $\beta_I = \beta_{II}$?
$y_{II} = X_{II}\beta_{II} + \varepsilon_{II}$

$\Leftrightarrow \begin{bmatrix} Y_I \\ Y_{II} \end{bmatrix} = \begin{bmatrix} \underset{44\times 4}{X_1} & \underset{44\times 4}{0} \\ \underset{32\times 4}{0} & \underset{32\times 4}{X_2} \end{bmatrix} \begin{bmatrix} \underset{4\times 1}{\beta_I} \\ \underset{4\times 1}{\beta_{II}} \end{bmatrix} + \begin{bmatrix} \varepsilon_I \\ \varepsilon_{II} \end{bmatrix}$ This is the unrestricted regression

$\Rightarrow$ 8 parameters

The original regression is the restricted regression
$\Rightarrow$ 4 parameters here

Clearly, there are 4 restrictions

$H_0: \beta_I = \beta_{II}$

$$F_0 = \left(\frac{n-k}{p}\right)\left(\frac{ESS_R - ESS_{UR}}{ESS_{UR}}\right) = \left(\frac{76-8}{4}\right)\left(\frac{18.48-(9.32+7.46)}{9.32+7.46}\right)$$

$$= 17\left(\frac{18.48-16.78}{16.78}\right) = 17(0.1013) = 1.7223$$

$c^*_{0.05, F(4,68)} = 2.5066$

Since $F_0 = 1.7223 \le 2.5066 = c^*_{0.05, F(4,68)}$, we fail to reject the null hypothesis that there was no structural change.

Question 2: $Invest_t = \beta_1 + \beta_2 t + \beta_3 RealGNP_t + \beta_4 int_r + \beta_5 infl_r$

a] Interpret the results.

$H_0: \beta = 0$ $\quad t_0 = \frac{\hat\beta_i}{se(\hat\beta_i)} \quad i = 1, \dots, 5$

b] $H_0: \beta_3 = 1$ $\quad t_0 = \frac{\hat\beta_3 - 1}{se(\hat\beta_3)} = \frac{0.6704 - 1}{0.0550} = -5.9927$

$\Rightarrow |t_0| = 5.9927 > 2.2282 = c^*_{0.05, t(15-5)}$

Reject H_0

c] $\hat I = \hat\beta_1 + \hat\beta_2 t + \hat\beta_3 GNP + \hat\beta_4 int + \hat\beta_5 infl$

Plug in $\hat\beta_1 = -0.5091$, $\hat\beta_2 = -0.0166$, $\hat\beta_3 = 0.6704$, $\hat\beta_4 = -0.0023$, $\hat\beta_5 = -9 \times 10^{-5}$

$nGNP = 3100$, $CPI = 212$, $infl = 2.12$, $int = 10$

$\Rightarrow RealGNP = \frac{nGNP}{CPI} = \frac{3100}{2.12} = 1.4623$

$t_{1983} = 16$

d] The $\hat\beta$'s are based on 1968-82 data. In part c), you are given $X_0 = [1 \;\; 16 \;\; \underline{3.1} \;\; 2.12 \;\; 10]$

or 1.4623 ?

See Lecture notes 3, pages 6 and 7

We don't know Y_0

(i) $E[Y_0 | X_0] = X_0 \beta$ - but this is infeasible

$Var(Y_0 | X_0) = \sigma^2$ - also infeasible

(ii) $\hat{E}[Y_0 | X_0, X] = X_0 \hat{\beta} \equiv \hat{Y}_0$

$V[\hat{Y}_0 | X_0, X] = V(X_0 \hat{\beta} | X_0, X) = X_0 V(\hat{\beta} | X_0, X) X_0'$

$= \sigma^2 X_0 (X'X)^{-1} X_0'$ - also infeasible

$\hat{V}[\hat{Y}_0 | X_0, X] = \hat{\sigma}^2 X_0 (X'X)^{-1} X_0'$

where $\hat{\sigma}^2 = \frac{1}{n-k} \hat{\varepsilon}' \hat{\varepsilon}$

Want CI for $\hat{Y}_0$: $\dfrac{\hat{Y}_0 - Y_0}{se(\hat{Y}_0 - Y_0)}$

$V(\hat{Y}_0 - Y_0 | X_0, X) = V[\hat{Y}_0 | X_0, X] + V[Y_0 | X_0, X] - \underbrace{2 Cov(\hat{Y}_0, Y_0 | X_0, X)}_{=0 \text{ since } Y_0 \perp\!\!\!\perp X}$

$= \sigma^2 X_0 (X'X)^{-1} X_0' + \sigma^2$ $\Leftrightarrow Y_0 \perp\!\!\!\perp X_0 \hat{\beta}$

$= \sigma^2 (1 + X_0 (X'X)^{-1} X_0')$ since $\hat{\beta} = f(X)$

$\Rightarrow \hat{Y}_0 - Y_0 \sim N(0, \sigma^2 (1 + X_0 (X'X)^{-1} X_0'))$

$\Rightarrow \dfrac{\hat{Y}_0 - Y_0}{\sqrt{\sigma^2 (1 + X_0 (X'X)^{-1} X_0')}} \sim N(0,1)$ Not feasible, though