STATA output

[1] What is t-stat of educ?

$$t = \frac{\hat\beta_{educ}}{se(\hat\beta_{educ})} = \frac{0.3046}{0.02841}$$

⋮

In STATA: $TSS = \Sigma(Y_i - \bar{Y})^2$
$MSS = \Sigma(\hat{Y}_i - \bar{Y})^2$
$RSS = \Sigma\underbrace{(\hat{Y}_i - Y_i)}_{\hat\varepsilon_i}^2$

$$\bar{R}^2 = 1 - \frac{n-1}{n-k}\frac{RSS}{TSS} = 1 - \frac{99}{96}\frac{RSS}{TSS}$$

$$F_0 = \frac{(SSR_R - SSR_U)/(p)}{SSR_u/(n-k)} = \frac{(TSS - RSS)/3}{RSS/96} = \frac{(702-97)/3}{97/96}$$

- $SSR_R = TSS$ since the restricted regression is $Y_i = \alpha + \varepsilon_i$ and $\hat\alpha_{OLS} = \bar{Y} \Rightarrow SSR_R = \Sigma(\underbrace{\hat{Y}_i}_{=\hat\alpha \cdot 1 = \bar{Y}} - Y_i)^2 = \Sigma(\bar{Y} - Y_i)^2 = TSS$.

[8] What is RMSE?

$$RMSE = \sqrt{\frac{1}{n-k}\Sigma\hat\varepsilon_i^2} = \sqrt{\frac{RSS}{n-k}} = \sqrt{\frac{96.79}{96}} \approx 1.004$$

also, $RMSE = \sqrt{RMS}$

MT99

Q1: b) Which coefficients are statistically significant?
for each $\hat\beta_j$, cf $\left|\frac{\hat\beta_i}{se(\hat\beta_i)}\right|$ with 1.96

c) $H_0: \beta_2 = \beta_3 \iff H_0: \beta_2 - \beta_3 = 0 \iff H_0: \Gamma\beta = 0 \iff H_0: [0\,0\,1\,-1\,0]\beta = 0$

$H_A: \beta_2 \neq \beta_3 \iff H_A: [0\,0\,1\,-1\,0]\beta \neq 0$

$$|t_0| = \left|\frac{\hat\beta_2 - \hat\beta_3}{se(\hat\beta_2 - \hat\beta_3)}\right| = \left|\frac{\Gamma\hat\beta - \gamma_0}{\sqrt{\hat\sigma^2\Gamma(X'X)^{-1}\Gamma'}}\right|$$

where $\hat\sigma^2 = \frac{1}{n-k}\sum_{i=1}^{n}\hat\varepsilon_i^2$

$C = \{(x_1, \dots, x_n): |t_0| \geq 1.96\}$ ← This is the critical region

We are missing $\hat\sigma^2$ and $(X'X)$

MT01

Q1: $r_t = \alpha + \beta r_t^* + e_t$ where r_t^* is the prediction of r_t

$H_0: \alpha = 0,\ \beta = 1 \iff H_0: \Gamma\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \gamma \iff H_0: \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}$

$$F = \frac{1}{p}(\Gamma\hat\delta - \gamma_0)'(\hat\sigma^2\Gamma(X'X)^{-1}\Gamma')(\Gamma\hat\delta - \gamma_0) \sim F(p, n-k)$$

$$= \frac{1}{p}(\hat\delta - \gamma_0)'(\hat\sigma^2(X'X)^{-1})(\hat\delta - \gamma_0)$$

$$= \frac{1}{p}(\hat\delta - \gamma_0)'\frac{(X'X)}{\hat\sigma^2}(\hat\delta - \gamma_0)$$

$$X'X = \begin{bmatrix}1 & \cdots & 1\\ r_1^* & \cdots & r_n^*\end{bmatrix}\begin{bmatrix}1 & r_1^*\\ \vdots & \vdots\\ 1 & r_n^*\end{bmatrix} = \begin{bmatrix}n & \Sigma r_t^*\\ \Sigma r_t^* & \Sigma r_t^{*2}\end{bmatrix}$$

$\Sigma(r_t^* - \bar r^*)^2 = \Sigma r_t^{2*} - 2\bar r^*\Sigma r_t^* + n\bar r^{*2}$

$52 = \Sigma r_t^{2*} - 2(10)(300) + 30(10)^2$

$\Sigma r_t^{*2} = 52 + 6000 - 3000 = 3052$

$= \Sigma r_t^{*2} - n\bar r^{*2}$

$\Rightarrow \Sigma r_t^{*2} = 52 + (30)(10)^2 = 3052$

Just plug in for the rest

MT01

Q2 a) $\underline{Y} = \underline{X}\beta + \varepsilon \Rightarrow R^2$
$\underline{Y} + c = \underline{X}\beta + \tilde{\varepsilon} \Rightarrow \tilde{R}^2$ } Is $R^2 = \tilde{R}^2$

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\underline{Y}' M_X \underline{Y}}{\underline{Y}' M^0 \underline{Y}} =$$

Let $\tilde{\underline{Y}} = \underline{Y} + c$

$$\tilde{R}^2 = 1 - \frac{\tilde{\underline{Y}}' M_X \tilde{\underline{Y}}}{\tilde{\underline{Y}}' M^0 \tilde{\underline{Y}}}$$

$$\tilde{\underline{Y}}' M_X \tilde{\underline{Y}} = (\underline{Y} + c)' M_X (\underline{Y} + c) = \underline{Y}' M_X \underline{Y} + \underbrace{c' M_X \underline{Y} + \underline{Y}' M_X c + c' M_X c}_{\substack{=0 \text{ if the original} \\ \text{regression includes a} \\ \text{constant.}}}$$

Similarly, we must check to see that $\tilde{\underline{Y}}' M^0 \tilde{\underline{Y}} = \underline{Y}' M^0 \underline{Y}$

a') Let $\tilde{\underline{Y}} = c\underline{Y}$

$$\Rightarrow \tilde{R}^2 = 1 - \frac{\tilde{\underline{Y}}' M_X \tilde{\underline{Y}}}{\tilde{\underline{Y}}' M^0 \tilde{\underline{Y}}} = 1 - \frac{c\underline{Y}' M_X \cdot c\underline{Y}}{c\underline{Y}' M^0 c\underline{Y}} = 1 - \frac{\underline{Y}' M_X \underline{Y}}{\underline{Y}' M^0 \underline{Y}}$$

c) $\underline{Y} = \underline{X}\beta + \varepsilon$
$\underline{Y} = \hat{\underline{X}}\hat{\beta} + \hat{\varepsilon}$ where $\hat{\underline{X}} = \underline{X} \underset{k \times k}{A}$

$$R^2 = 1 - \frac{\underline{Y}' M_X \underline{Y}}{\underline{Y}' M^0 \underline{Y}}$$

$$\hat{R}^2 = 1 - \frac{\underline{Y}' M_{\hat{X}} \underline{Y}}{\underline{Y}' M^0 \underline{Y}} = 1 - \frac{\underline{Y}' M_X \underline{Y}}{\underline{Y}' M^0 \underline{Y}} = R^2$$

$$\begin{aligned} M_{\hat{X}} &= I - \hat{\underline{X}}(\hat{\underline{X}}'\hat{\underline{X}})^{-1}\hat{\underline{X}}' \\ &= I - \underline{X}A(A'\underline{X}'\underline{X}A)^{-1}A'\underline{X}' \\ &= I - \underline{X}A(A)^{-1}(\underline{X}'\underline{X})^{-1}(A')^{-1}A'\underline{X}' \\ &= I - \underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}' = M_X \end{aligned}$$