Office hour: Mondays from 2:30-3:30 in Alper Room

## OLS in matrix notation

scalar model: $y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_k X_{ki} + \varepsilon_i \quad , i=1,\dots,n$

↳ Everything here is a scalar

vector model: $y_i = X_i \beta + \varepsilon_i$ where $\overset{1\times k}{X_i} \equiv [X_{1i} \cdots X_{ki}]$
$i=1,\dots,n$ $\quad \underset{k\times 1}{\beta} = [\beta_1 \cdots \beta_k]'$

matrix model: $Y = X\beta + \varepsilon$ where $\underset{n\times k}{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}$; $\underset{n\times 1}{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$; $\underset{n\times 1}{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$

OLS problem:

$$\hat{\beta} \equiv \underset{b}{\text{argmin}} \sum_{i=1}^{n} (y_i - X_i b)^2 = \text{argmin} \overbrace{\underbrace{(Y - Xb)'}_{1\times n} \underbrace{(Y - Xb)}_{n\times 1}}^{1\times 1}$$

$$= \underset{b}{\text{argmin}} (Y' - b'X')(Y - Xb)$$

$$= \underset{b}{\text{argmin}} (Y'Y - b'X'Y + b'X'Xb - Y'Xb)$$

$$= \underset{b}{\text{argmin}} \underbrace{(\underset{1\times n}{Y'}\underset{n\times 1}{Y} - 2\underset{1\times k}{Y'}\underset{k\times n}{X}\underset{n\times 1}{b} + \underset{1\times k}{b'}\underset{k\times n}{X'}\underset{n\times k}{X}\underset{k\times 1}{b})}_{1\times 1}$$

Rules for matrix calculus:

$$\frac{\partial (a'x)}{\partial x} = a \quad ; \quad \frac{\partial x'Ax}{\partial x} = (A + A')x$$

Thus, the FOCs are:

$$(b): \quad -2X'Y + (X'X + (X'X)')\hat{\beta} = 0$$

$$\Rightarrow \quad -2X'Y + 2X'X\hat{\beta} = 0$$

$$\Rightarrow \quad X'X\hat{\beta} = X'Y$$

$$\Rightarrow \quad \hat{\beta} = (X'X)^{-1}X'Y$$