

Suppose you are a firm and are trying to decide how much research and development to undertake. The profit function for firms in your industry takes the form:

$$\Pi_i = \beta_1 + \beta_2 RD_i + \beta_3 RD_i^2 + u_i$$

where  $\Pi_i$  and  $RD_i$  are profits and research and development (R&D) expenditures for firm  $i$ , and where  $\beta_2 > 0$  and  $\beta_3 < 0$ . Given data on profits and R&D expenditures for firms in your industry explain how you would use them to choose the optimal level of R&D.

**PROBLEM 4 (15 points):**

Consider the following regression model:

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + v_i$$

Explain in as much detail as you can how you would test for the following hypotheses:

- (i)  $\beta_2 = \beta_3$
- (ii)  $\beta_3 = 3\beta_4$
- (iii)  $\beta_2 = \beta_3 = \beta_4 = 0$

**PROBLEM 5 (30 points):**

Suppose you have annual data from 50 different states over the last 20 years and you are trying to estimate the effect of different policies on the local unemployment rate. You run two regressions and obtain the following output:

Variable	Coefficient	Standard Error
Constant	0.013	0.05
Average Wage	-0.025	0.08
Minimum Wage	0.02	0.04
Average Welfare Benefits	0.031	0.012
Observations	1000	
$R^2$	0.114	

Variable	Coefficient	Standard Error
Constant	0.013	0.05
Average Wage	-0.035	0.06
Observations	1000	
$R^2$	0.103	

Carry the following tests at the 10% significance level assuming that the assumptions of the classical normal regression model hold.

- (i) Test whether the value of minimum wage influences unemployment.
- (ii) Test whether welfare benefits increases unemployment.
- (iii) Test jointly whether the minimum wage and welfare benefits influence unemployment.
- (iv) Describe how you would test whether the effect of welfare benefits on unemployment is constant over time.
- (v) Describe how you would test whether the effect of minimum wage is constant over states.
- (vi) Describe how you would test for the hypothesis whether the effect of minimum wage depends on average welfare benefits.

**Economics 203B**  
**Winter 2001**  
**Midterm Examination**  
**Ekaterini Kyriazidou**

You have 1 hour and 20 minutes to do the following three problems. Write carefully and clearly. Try to answer ALL questions as partial credit will be given. GOOD LUCK!

**PROBLEM 1 (15 points):**

One aspect of the rational expectations hypothesis involves the claim that expectations are unbiased, that is, that the average prediction is equal to the observed realization of the variable under investigation. This claim can be tested by reference to announced predictions and actual values of the rate of interest on 3-month U.S. Treasury Bills published in the *Goldsmith-Nagan Bond and Money Market Letter*. The results of Least Squares estimation (based on 30 quarterly observations) of the regression of the actual on the predicted interest rates were as follows:

$$T\beta = \delta_0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$r_t = 0.24 + 0.94r_t^* + \hat{\epsilon}_t$$

where  $r_t$  is the observed interest rate and  $r_t^*$  is the average expectation of  $r_t$  held at the end of the preceding quarter. The sample data give

$$(Y'X) = \begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \quad \sum_t r_t^* = 300 \quad \sum_t (r_t^* - \bar{r}^*)^2 = 52 \quad RSS = 28.56$$

where *RSS* stands for Residual Sum of Squares. Carry out a test (at the 5% significance level) of the rational expectations hypothesis using the results above, assuming that all basic assumptions of the classical normal regression model hold.

**PROBLEM 2 (30 points):**

For each one of the following claims show whether they are true or false.

$$\frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2}$$

- (a) The  $R^2$  of a  $k$ -variate regression does not change if we add to the dependent variable a constant and/or if we multiply the dependent variable by a constant.
- (b) The residuals from the regression of  $Y$  on  $X_1$  and  $X_2$  coincide with the residuals from the "residual" regression of  $Y$  on  $\bar{X}_1$  where  $\bar{X}_1$  are the residuals from the regression of  $X_1$  on  $X_2$ .
- (c) For the  $k$ -variate regression model,  $y = X\beta + \epsilon$ , the fit as measured by  $R^2$  does not change if we transform the  $X$  matrix by postmultiplying it by a  $k \times k$  non-singular matrix.

**PROBLEM 3 (10 points):**