ECON 203B Winter 2005 Ekaterini Kyriazidou

PROBLEM SET 3

PROBLEMS DUE FRIDAY 2/11

Do the following problems:

Problem 1:

The regression slope $\hat{\beta}$ in a CNLR model is distributed $N\left(\beta, \sigma^2_{\hat{\beta}}\right)$ where $\sigma^2_{\hat{\beta}} = 1$. The null hypothesis $\beta = 0$ will be tested at the 10% significance level by using the statistic $Z_0 = \hat{\beta}/\sigma_{\hat{\beta}}$. That is, the null will be rejected if and only if $|Z_0| > 1.645$.
(a) Write and run a program that tabulates the power of the test at the following 9 values of the true parameter $\beta :\ -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2.$
(b) Redo (a) for the situation where $\sigma^2_{\hat{\beta}} = 4$.
(c) What do your two tables tell you about the effect of $\sigma^2_{\hat{\beta}}$ on the power of the test?

Problem 2:

The regression slopes $\hat{\beta}_1$ and $\hat{\beta}_2$ in a CNLR model are distributed as bivariate normal:

$$\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix} \right)$$

where $r = 0.6$. The joint null hypothesis $\beta_1 = \beta_2 = 0$ will be tested at the 5% significance level by using the statistic

$$W_0 = \frac{\left(\hat{\beta}_1^2 + \hat{\beta}^2 - 2r\hat{\beta}_1\hat{\beta}_2\right)}{(1 - r^2)}$$

That is, the null will be rejected if and only if $|W_0| > 5.99$.
(a) Write and run a program that tabulates the power of the test at the following 9 pairs of the true parameter vector $(\beta_1, \beta_2) :\ (-1, 1), (-1, 0), (-1, -1), (0, 1), (0, 0), (0, -1), (1, 1), (1, 0), (1, -1)$.
(b) Redo (a) for the situation where $r = -0.6$.
(c) What do your two tables tell you about the effect of the correlation r on the power of the test?

Problem 3:

Suppose that Y_i is a discrete random variable (actually a *count variable,* such as, for example, number of accidents) whose conditional distribution given X_i is

$$\Pr\left(Y_i = y_i | X_i = x_i\right) = \frac{e^{-\beta x_i} (\beta x_i)^{y_i}}{y_i!}, \qquad y_i = 0, 1, 2, ...$$

where X_i is a positive scalar random variable and $\beta > 0$. Assume that we have n independent observations $\{(Y_i, X_i)\}_{i=1}^n$.

(a) Write down the (conditional) log-likelihood function of the sample and compute the maximum likelihood estimator of β, $\hat{\beta}_{MLE}$.

(b) Find the asymptotic distribution of $\hat{\beta}_{MLE}$.

(c) Find the exact distribution of $\hat{\beta}_{MLE}$. (HINT: The sum of independent Poisson random variables with parameters λ_j is a Poisson variable with parameter $\sum_j \lambda_j$.)

Problem 4:

In the standard CNLR model,

$$Y_i = X_i\beta + \varepsilon_i$$

where $\varepsilon|X \sim N\left(0, \sigma^2 I_n\right)$, assume that $K = 1$ and that $\sigma^2 = \beta^2$. Obtain the maximum likelihood estimator for β and the Cramér-Rao lower bound.

Problem 5:
Derive the log-likelihood function, the first order conditions for maximization, and the information matrix for the model:

$$\begin{aligned} Y_i &= X_i\beta + \varepsilon_i \\ \varepsilon_i|X_i &\sim N\left(0, (Z_i\gamma)^2\right) \end{aligned}$$

assuming i.i.d. sampling of (Y_i, X_i) across individuals. Here Z_i is a $1 \times r$ subvector of X_i.

Problem 6:
Five sample observations are

X	4	1	5	8	2
Y	6	3	12	15	4

Assume a linear model, $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$, with heteroskedasticity of the form $Var\left(Y_i\right) \equiv Var\left(\varepsilon_i\right) \equiv \sigma_i^2 = \sigma^2 X_i^2$ where σ^2 is a positive constant. Calculate the OLS and GLS estimates of β_1 and β_2 and the corresponding standard errors.

Problem 7:
Determine whether the following statement is true or false: Suppose that the CLR model applies to

$$E\left(Y|X\right) = X\beta$$

that T is a nonstochastic nonsingular matrix and that $Y^* = TY$ and $X^* = TX$; then the GLS regression of Y^* on X^* gives the same coefficient estimates as OLS of Y on X.