Section 4.1   Differentiable Functions

Theorem 4.1.1

If f is differentiable at x, then f is continuous at x.

Theorem 4.1.2

Suppose that f and g are differentiable at x. Then,

(a) For any constants α and β, $\alpha f + \beta g$ is differentiable at x and

$$(\alpha f + \beta g)' = \alpha f' + \beta g'.$$

(b) The product fg is differentiable at x and $(fg)' = f'g + fg'$.

(c) If $g(x) \neq 0$, then f/g is differentiable at x and

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

Theorem 4.1.3 (The Chain Rule)

Suppose that f is differentiable at x and g is differentiable at $f(x)$. Then $g \circ f$ is differentiable at x and $(g(f(x))' = g'(f(x))f'(x)$

Section 4.1   Differentiable Functions

Theorem 4.2.1

Suppose that f is continuous on the finite interval $[a,b]$. Let c be a point where f attains its maximum. If $a < c < b$ and f is differentiable at c, then $f'(c) = 0$.

Theorem 4.2.2 (Rolle's Theorem)

Suppose that f is continuous on the finite interval $[a,b]$, differentiable on (a,b), and $f(a) = 0 = f(b)$. Then there is a point c satisfying $a < c < b$ such that $f'(c) = 0$

Theorem 4.2.3 (The Mean Value Theorem)

Suppose that f is continuous on the finite interval $[a,b]$ and differentiable on (a,b). Then there is a point c satisfying $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem 4.2.4 (The Fundamental Theorem, Part I)

Let f be a continuously differentiable function on a finite interval $[a,b]$. Then,

$$\int_a^b f'(x)dx = f(b) - f(a).$$

Theorem 4.2.5 (The Fundamental Theorem, Part II)

Let f be a continuous function on a finite interval $[a,b]$. Define

$$F(x) \equiv \int_a^x f(t)dt \, .$$

Then F is continuously differentiable on $[a,b]$, and $F'(x) = f(x)$.