Section 1.1 Preliminaries

Proposition 1.1.1

(a) For all real numbers x, y, and z, if $x + z = y + z$, then $x = y$.
(b) Additive inverses are unique.
(c) For all real numbers x and y, $(-x)(-y) = xy$.
(d) For all real numbers x and y, if $x \le y$, then $-x \ge -y$.

Proposition 1.1.2

(a) $|x| \ge 0$ for all x and $|x| = 0$ if and only if $x = 0$.
(b) $|xy| = |x||y|$ for all x and y.
(c) $|x + y| \le |x| + |y|$ for all x and y (the triangle inequality).

Section 1.2 Sets and Functions

Section 1.3 Cardinality

Proposition 1.3.1

Let S, T, and U be sets. If S and T have the same cardinality and T and U have the same cardinality, then S and U have the same cardinality.

Proposition 1.3.2

If S is an infinite subset of a countable set T, then S is countable.

Theorem 1.3.3 (The Fundamental Theorem of Arithmetic)

Every positive integer $N \ge 2$ can be written uniquely as a finite product of positive integral powers of primes:

$$N = p_1^{s_1} p_2^{s_2} \ldots p_n^{s_n}.$$

Proposition 1.3.4

If S and T are countable sets, then $S \times T$ is countable.

Theorem 1.3.5

The set of rational numbers, $\mathbf{Q}$ is countable.

Theorem 1.3.6

The set of real numbers between 0 and 1, that is, $(0,1)$, is not countable.

Section 1.4 Methods of Proof

Proposition 1.4.1

Suppose that and $x > 0$. Then $x > 1$.

Proposition 1.4.2

$\sqrt{2}$ is irrational.

Proposition 1.4.3

If n is a positive integer, then

$$1 + 2 + ... + n = \frac{n(n+1)}{2}.$$

Theorems from Lecture

Theorem

Let A an arbitrary set. Denote by $P(A)$ the set of all subsets of A. ie. $P(A) = \{B \mid B \subset A\}$. Then A and $P(A)$ do not have the same cardinality.