6.4.1 Find the radius of convergence of the series $\sum a_j x^j$ for each of the following choices of the coefficients a_j :

6.4.1.c $\ln j^3$

Answer to exercise 6.4.1.c:

Using Mathematica,

Limit[Abs[Log[j^3]]^(1/j), j → ∞] **=1**

The limit exists, so by corollary 6.1.2, we have $\alpha = \limsup \left|\ln j^3\right|^{1/j} = 1$.

Therefore, $R = \frac{1}{\alpha} = \frac{1}{1} = 1$.

6.4.1.d j^3

Answer to exercise 6.4.1.d:

Using Mathematica,

Limit[Abs[j^3]^(1/j), j → ∞] **=1**

The limit exists, so by corollary 6.1.2, we have $\alpha = \limsup \left|j^3\right|^{1/j} = 1$.

Therefore, $R = \frac{1}{\alpha} = \frac{1}{1} = 1$.

6.4.1.e $\frac{\sin j}{j}$

Answer to exercise 6.4.1.e:

Using Mathematica,

Limit[Abs[Sin[j]/j]^(1/j), j → ∞] =1

The limit exists, so by corollary 6.1.2, we have $\alpha = \limsup \left|\frac{\sin j}{j}\right|^{1/j} = 1$.

Therefore, $R = \frac{1}{\alpha} = \frac{1}{1} = 1$.

6.4.1.f $2j+1$

Using Mathematica,

Limit[Abs[2 j + 1]^(1/j), j → ∞] **=1**

The limit exists, so by corollary 6.1.2, we have $\alpha = \limsup \left|2j+1\right|^{1/j} = 1$.

Therefore, $R = \frac{1}{\alpha} = \frac{1}{1} = 1$.

6.4.1.g 2^{j^2}

Using Mathematica, we see that

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Limit[Abs[2^(j^2)]^(1/j), j → ∞]
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$=\infty$

Therefore, the sequence $\left|2^{j^2}\right|^{1/j}$ is unbounded. Therefore, $\alpha = \limsup \left|2^{j^2}\right|^{1/j} = \infty$.

By definition, we have that $R = 0$.