

5.6.7 Let  $(M, \rho)$  be a metric space and suppose that  $\{x_n\}$  is a sequence in  $(M, \rho)$  so that  $\lim_{n \rightarrow \infty} x_n = x$  and  $\lim_{n \rightarrow \infty} x_n = y$ . Prove that  $x = y$ .

Proof of exercise 5.6.7:

Since  $\lim_{n \rightarrow \infty} x_n = x$ , we have that  $\rho(x_n, x) \rightarrow 0$  as  $n \rightarrow \infty$ .

Since  $\lim_{n \rightarrow \infty} x_n = y$ , we have that  $\rho(x_n, y) \rightarrow 0$  as  $n \rightarrow \infty$ .

We also know by the properties of a metric that:

$\rho(x, y) \leq \rho(x_n, x) + \rho(x_n, y)$ . Taking limits, we have that:

$\rho(x, y) \rightarrow 0$  as  $n \rightarrow \infty$ . i.e.  $x = y$ .