

3.1.9 Let  $f(x) = 3x^3 - 2$  and let  $\varepsilon > 0$  be given. Find a  $\delta$  so that  $|x-1| \leq \delta \Rightarrow |f(x)-1| \leq \varepsilon$ .

Answer to exercise 3.1.9:

Let  $|x-1| \leq \delta \Rightarrow |x|-|1| \leq \delta$  by exercise 1.1.10  $\Rightarrow |x| \leq \delta+1$

$$\begin{aligned} |f(x)-1| &= |3x^3 - 2 - 1| = |3x^3 - 3| = 3|x^3 - 1| = 3|x-1||x^2 + x + 1| \\ &\leq 3\delta|\delta^2 + 2\delta + 1 + \delta + 1 + 1| = 3\delta|\delta^2 + 3\delta + 3| \end{aligned}$$

Take  $\delta \leq \min\left\{\frac{\varepsilon}{21}, 1\right\} \Rightarrow |f(x)-1| \leq 3\delta|1+3+3| = 21\delta \leq 21\frac{\varepsilon}{21} = \varepsilon$ .