

2.5.4 Prove that least upper bounds are unique. That is, if  $\mu_1$  and  $\mu_2$  are both least upper bounds for the set  $S$ , then  $\mu_1 = \mu_2$ .

Proof of exercise 2.5.4:

Let  $S$  be a bounded set. Suppose  $\mu_1$  and  $\mu_2$  are both least upper bounds for  $S$ . Since  $\mu_1$  is a least upper bound for  $S$  and since  $\mu_2$  is an upper bound for  $S$ , by definition,  $\mu_1 \leq \mu_2$ . Since  $\mu_2$  is a least upper bound for  $S$  and since  $\mu_1$  is an upper bound for  $S$ , by definition,  $\mu_2 \leq \mu_1$ . Thus, by axiom (O2), we have that  $\mu_1 = \mu_2$ . i.e. least upper bounds are unique. Q.E.D.