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Problem Set 4

2.5.2 Let *S* be a set of real numbers and suppose that a_n is an upper bound for S for each *n*. Prove that if $a_n \to a$, then *a* is an upper bound for *S*.

Lemma 2.5.2.a:

If $a < b$, then $\exists \delta > 0 \ni a + \delta < b$.

Proof of lemma 2.5.2.a:

Suppose $a < b$. Then $b - a > 0$. We also know that $b - a > \frac{b-a}{2} > 0$

Take $\delta = \frac{b-a}{2}$. Then $b - a > \delta \Rightarrow a + \delta < b$. Q.E.D.

Proof of exercise 2.5.2:

In order to get a contradiction, assume that a is not an upper bound for *S*. That is, $\exists\, x \in S \ni x > a$.

By lemma 2.5.2.a, $\exists \delta > 0 \ni a + \delta < x$.

Since $a_n \to a$, $\forall \varepsilon > 0 \; \exists\, N(\varepsilon) \in \mathbf{N} \ni \forall\, n \geq N$, $|a_n - a| < \varepsilon$. Take $\varepsilon = \delta$. Then

$\exists\, N(\delta) \in \mathbf{N} \ni \forall\, n \geq N$, $|a_n - a| < \delta$

$\Rightarrow -\delta < a_n - a < \delta \Rightarrow a - \delta < a_n < a + \delta$

In particular, $a_n < a + \delta < x$. Therefore, $\exists\, x \in S \ni x > a_n$ for some *n*, which is a contradiction. $\to\leftarrow$ Therefore, a must be an upper bound for *S*. Q.E.D.