2.2.4 Let $\{b_n\}$ be a bounded sequence and suppose $a_n \to 0$. Prove that $a_n b_n \to 0$.

Proof of exercise 2.2.4:

We know that $\{b_n\}$ is bounded. That is, $\exists\, M > 0 \ni \forall n \in \mathbf{N},\ |b_n| < M$.

We also know that $a_n \to 0$. That is, $\forall \varepsilon > 0, \exists\, N(\varepsilon) \in \mathbf{N} \ni \forall\, n \geq N,\ |a_n| < \dfrac{\varepsilon}{M}$.

Take $\varepsilon > 0$ arbitrary. Then, $\forall n \geq N$, we have:

$|a_n b_n| = |a_n||b_n| \leq |a_n| M \leq \dfrac{\varepsilon}{M} M = \varepsilon$ where proposition 1.1.2.b was used in the first step. That is, $a_n b_n \to 0$. Q.E.D.