2.1.7 Suppose that $a_n \to a$ and $a_n \geq b$ for each n. Prove that $a \geq b$.

Proof of exercise 2.1.7:

$a_n \to a$ as $n \to \infty \Rightarrow \forall \varepsilon > 0 \; \exists \; N \in \mathbf{N} \ni \forall n \geq N, \; |a_n - a| \leq \varepsilon.$

In order to get a contradiction, suppose $a < b$. Then, by lemma 2.1.5.a,
$\exists \delta > 0 \ni a + \delta < b.$

Take $\varepsilon = \delta \Rightarrow \exists N \in \mathbf{N} \ni \forall n \geq N, \; |a_n - a| \leq \delta$
$\Rightarrow -\delta \leq a_n - a \leq \delta \Rightarrow a - \delta \leq a_n \leq a + \delta < b.$ In particular, $\forall n \geq N, \; a_n < b$,
which is a contradiction. $\rightarrow\leftarrow$ Q.E.D.