2.1.4 Prove directly that each of the following sequences diverges to $+\infty$ or $-\infty$:

2.1.4.d $a_n = \frac{n!}{2^n}$

Want to show $\forall M > 0, \exists\ N(M) \in \mathbf{N} \ni \forall n \geq N, \frac{n!}{2^n} \geq M$.

$$\frac{n!}{2^n} = \left(\frac{n}{2}\right)\left(\frac{n-1}{2}\right)\ldots\left(\frac{3}{2}\right)\left(\frac{2}{2}\right)\left(\frac{1}{2}\right) > \left(\frac{n}{2}\right)(1)\ldots(1)(1)\left(\frac{1}{2}\right) = \frac{n}{4}.$$

Take $M > 0$ arbitrarily large. $\frac{n}{4} \geq M$ whenever $n \geq N \Leftrightarrow n \geq 4M$ whenever $n \geq N$. Choose $N \geq 4M$

Then we have $\forall n \geq N$,

$\frac{n!}{2^n} \geq \frac{n}{4} \geq M$. Thus, $\frac{n!}{2^n} \to +\infty$ as $n \to \infty$. Q.E.D.