1.4.ex.2 Let $\{x_n\}$ be a sequence of numbers such that $x_1 = 2,\ x_2 = 4,$ and such that $x_{n+2} = 4x_{n+1} - 4x_n\ \forall n \geq 1$. Show by using induction that $x_n = 2^n$ for each $n \geq 1$.

Proof of exercise 1.4.ex.2:

Proof by induction with (n=n): $x_n = 2^n, x_{n+1} = 2^{n+1}$.

(n=1): $x_1 = 2^1 = 2,\ x_2 = 2^2 = 4$. Thus, (n=1) is true.

Assume (n=n): $x_n = 2^n, x_{n+1} = 2^{n+1}$

(n=n+1): $x_{n+1} = 2^{n+1}, x_{n+2} = 2^{n+2}$

The first part: $x_{n+1} = 2^{n+1}$ holds by the induction assumption.

$x_{n+2} = 4x_{n+1} - 4x_n$ by assumption. Thus, $x_{n+2} = 4x_{n+1} - 4x_n = 4(2^{n+1}) - 4(2^n)$ by induction assumption.

$\Rightarrow x_{n+2} = 2^2 2^{n+1} - 2^2 2^n = 2(2^{n+2}) - 1(2^{n+2}) = (2-1)2^{n+2} = 2^{n+2}$. Thus, (n=n+1) holds. Q.E.D.