1.4.9 Let $x > -1$ and n be a positive integer. Prove Bernoulli's inequality:

$$(1+x)^n \geq 1+nx.$$

Proof of exercise 1.4.9:

Since we are dealing only with positive integers, we may use mathematical induction. (Standard induction techniques will not work on the set of all integers.)

Since $x > -1$, $(1+x) > 0$.

(n=1): $(1+x)^1 = 1+x \geq 1+x = 1+1x.$ Thus, (n=1) holds.

Assume (n=n): $(1+x)^n \geq 1+nx.$

(n=n+1): $(1+x)^{n+1} = (1+x)^n(1+x) \geq (1+nx)(1+x) = 1+x+nx+nx^2.$

Since $n > 0$ by assumption and $x^2 > 0$ by problem set 1, we have:

$1+x+nx+nx^2 \geq 1+x+nx = 1+(n+1)x.$

Thus, $(1+x)^{n+1} \geq 1+(n+1)x.$ i.e. (n=n+1) holds. Q.E.D.