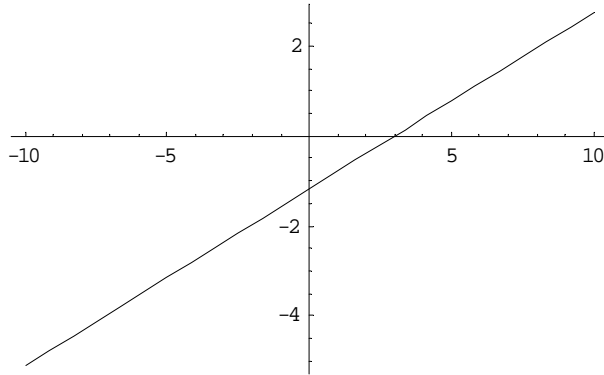


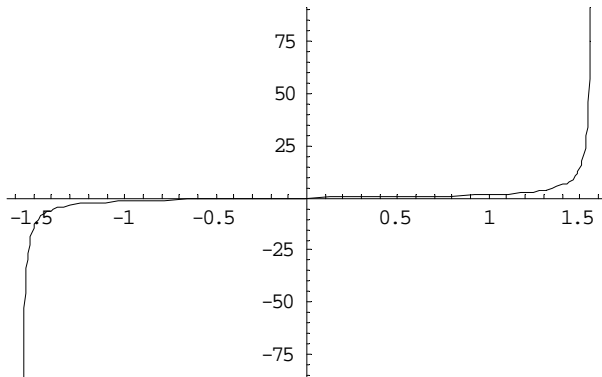
1.3.7 Find an explicit one-to-one correspondence between the interval  $(-1,7)$  and the real numbers  $\mathbf{R}$ .

Answer to exercise 1.3.7:

Let  $g : (-1,7) \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$  be defined by  $g(x) = \frac{\pi}{8}(x-3)$ . Clearly,  $g$  is a one-to-one correspondence between  $(-1,7)$  and  $(-\frac{\pi}{2}, \frac{\pi}{2})$ :



Let  $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbf{R}$  be defined by  $f(x) = \tan x$ . On this domain,  $f$  is a one-to-one correspondence between  $(-\frac{\pi}{2}, \frac{\pi}{2})$ :



Then, we have that  $f \circ g : (-1,7) \rightarrow \mathbf{R}$  defined by  $f \circ g(x) = \tan[\frac{\pi}{8}(x-3)]$  is a one-to-one correspondence between  $(-1,7)$  and  $\mathbf{R}$ :

