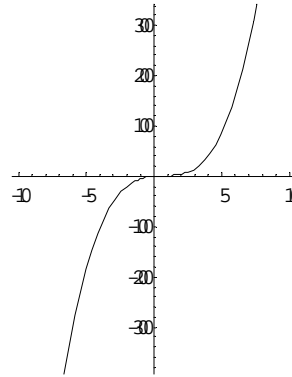
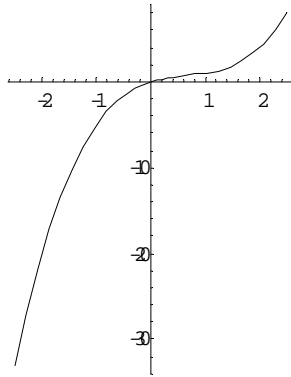


1.2.extra.2 Let f be a function defined on the real numbers with values in the real numbers such that $f \circ f(x) = x^3 - 2x^2 + 2x$ for each x real, and such that $f(0) = 0$. Show that f is injective. Find the value of $f(1)$.

Proof of exercise 1.2.extra.2:

$$f \circ f(x) = x^3 - 2x^2 + 2x:$$



Clearly, this function is injective. Thus, by 1.2.extra.1, since $f \circ f$ is injective, it follows that f is also injective.

$$f \circ f \circ f(1) = f(1)^3 - 2f(1)^2 + 2f(1) \quad \text{by definition of } f \circ f(x)$$

$$f \circ f \circ f(1) = f \circ (1^3 - 2(1)^2 + 2(1)) = f(1) \quad \text{by definition of } f \circ f(x)$$

$$\Rightarrow f(1) = f(1)^3 - 2f(1)^2 + 2f(1) \quad \text{transitivity of “=”}$$

$$\Rightarrow 0 = f(1)^3 - 2f(1)^2 + f(1)$$

$$\Rightarrow 0 = f(1)[f(1)^2 - 2f(1) + 1]$$

$$\Rightarrow 0 = f(1)[f(1) - 1][f(1) - 1]$$

Thus, either $f(1) = 0$ or $f(1) - 1 = 0 \Rightarrow f(1) = 1$

Since $f(0) = 0$ and since f is injective, it follows that $f(1) \neq 0$. Thus, it must be the case that $f(1) = 1$