

1.1.ex.1 Prove that, $\forall x \in \mathbf{R}, x^2 \geq 0$

Proof of extra exercise for 1.1:

Case I: $x \geq 0$

$$x \geq 0 \Rightarrow xx \geq 0x$$

$$x^2 = xx \geq x(0) = 0$$

Axiom O5 and defn of “ \geq ”

Defn of “ 2 ” and lemma

1.1.10.a

Case II: $x < 0$

$$-x > -0 = 0$$

Proposition 1.1.1.d and

Lemma 1.1.5.a

$$x^2 = xx = (-x)(-x)$$

Defn of “ 2 ” and proposition

1.1.1.c

$$(-x) > 0 \Rightarrow (-x)(-x) > 0(-x)$$

Axiom O5 and defn of “ $>$ ”

$$\Rightarrow x^2 = (-x)(-x) > 0(-x) = 0$$

Lemma 1.1.5.a