1.1.5 Use part (d) of Proposition 1.1.1 to prove that if $x \le y$ and $z \le 0$, then $zy \le zx$.

Lemma 1.1.5.a: $-0 = 0$

Proof of Lemma 1.1.5.a:

$0 + 0 = 0$	Additive identity
$0 + (-0) = 0$	Additive inverse
$\Rightarrow 0 + 0 = 0 + (-0)$	
$\Rightarrow 0 = -0$	Proposition 1.1.1.a

Lemma 1.1.5.b: $-(-x) = x$

Proof of Lemma 1.1.5.b:

$x + (-x) = 0$	Additive inverse
$(-x) + (-(-x)) = 0$	Additive inverse
$\Rightarrow x + (-x) = (-x) + (-(-x))$	
$\Rightarrow x + (-x) = (-(-x)) + (-x)$	Additive commutativity
$\Rightarrow x = (-(-x))$ or $-(-x) = x$	Proposition 1.1.1.a

Proof of Exercise 1.1.5:

$z \le 0 \Rightarrow -z \ge -0$	Proposition 1.1.1.d
$\Rightarrow -z \ge -0 = 0$	Lemma 1.1.5.a
$x \le y \Rightarrow (-z)x \le (-z)y$	Property O5
$\Rightarrow -(-z)x \ge -(-z)y$	Proposition 1.1.1.d
$\Rightarrow zx \ge zy$	Lemma 1.1.5.b
$\Rightarrow zy \le zx$	Definition of "$\ge$" Q.E.D.