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Problem Set #1

1.1.2 Use the field properties of the real numbers to provide a careful proof of the elementary algebraic identity  $(x + y)^2 = x^2 + 2xy + y^2$ .

Proof of exercise 1.1.2:

$$(x + y)^2 = (x + y)(x + y)$$

$$(x + y)(x + y) = (x + y)x + (x + y)y$$

$$(x + y)x + (x + y)y = xx + yx + xy + yy$$

$$xx + yx + xy + yy = xx + xy + xy + yy$$

$$xx + xy + xy + yy = xx + 1xy + 1xy + yy$$

$$xx + 1xy + 1xy + yy = xx + (1 + 1)xy + yy$$

$$xx + (1 + 1)xy + yy = xx + 2xy + yy$$

$$xx + 2xy + yy = x^2 + 2xy + y^2$$

$$\text{Therefore, } (x + y)^2 = x^2 + 2xy + y^2$$

Definition of  $^2$

Distributive property

Distributive property

Multiplicative commutativity

Multiplicative identity

Distributive property

Define  $2 \equiv 1 + 1$

Defintion of  $^2$

Q.E.D.