

1.1.11 Let  $a$  and  $b$  be real numbers with  $a < b$ . Prove that there are integers  $m$  and  $n \neq 0$

so that  $a < \frac{m}{n} < b$

Proof:  $a < b \Rightarrow b - a > 0$

Definition of positivity

$\Rightarrow \exists n \in \mathbb{N} \exists n(b - a) > k \quad \forall k \in \mathbb{R}, n \neq 0$

Archimedean property

$\Rightarrow nb - na > 1$

Distributive property

$\Rightarrow nb > 1 + na > na$

Define  $m \equiv$  smallest integer less than or equal to  $1 + na$

$\Rightarrow nb > m > na$

$\Rightarrow a < \frac{m}{n} < b$

Since  $n \neq 0$