3 Prove by induction that $n^n \ge n!$.

Lemma 3:

$$(a+1)^n \ge a^n \quad \forall a > 0$$

Proof of lemma 3: Induct on *n*.

$(n=1):(a+1)^1 = a+1 \ge a = a^1$. Thus, $(n=1)$ is true.

Assume $(n=n)$ is true: $(a+1)^n \ge a^n$.

$(n=n+1):(a+1)^{n+1} = (a+1)^n(a+1) \ge a^n(a+1) = a^{n+1} + a \ge a^{n+1}$. Q.E.D.

Proof of question 3: Induct on *n*.

$(n=1):1^1 = 1 \ge 1 = 1!$ Thus, $(n=1)$ is true.

Assume $(n=n)$ is true: $n^n \ge n!$

$(n=n+1):(n+1)^{n+1} = (n+1)^n(n+1) \ge n^n(n+1) \ge n!(n+1) = (n+1)!$

where lemma 3 was used in the second step and the induction assumption was used in the third step. Q.E.D.