1.a Show by using the axioms of the real numbers that $z^2 \geq 0$ for each real z.

Proof of question 1.a:

It is necessary to break this proof up into two cases: I) $z \geq 0$ and II) $z < 0$.

Case I: $z \geq 0$

$z \geq 0 \Rightarrow zz \leq 0z$	Axiom O5

Claim 1: $0z = 0$

Proof of claim 1:

$0z + 0 = 0z$	Axiom P3
$0z = (0+0)z$	Axiom P3
$(0+0)z = z(0+0)$	Axiom P5
$z(0+0) = z0 + z0 = 0z + 0z$	Axioms P9 and P5
$\Rightarrow 0z + 0 = 0z + 0z$	Transitivity
$\Rightarrow 0z = 0$	Proposition 1.1.1.a
Thus, we have: $z^2 \equiv zz > 0z = 0$.	Q.E.D.

Case II: $z < 0$

$z < 0 \Rightarrow -z > 0$	Proposition 1.1.1.d
$-z > 0 \Rightarrow (-z)(-z) > 0(-z)$	Axiom O5
$(-z)(-z) = zz = z^2$	Proposition 1.1.1.c
$0(-z) = 0$	Claim 1
$\Rightarrow z^2 = (-z)(-z) > 0(-z) = 0$	Transitivity
$\Rightarrow z^2 \geq 0$	Q.E.D.

1.b Define $2 \equiv 1+1$, and use the notation x^2 to denote the multiplication $x \cdot x$. Show by using the axioms of the real numbers and part (a), that $2 \cdot (x \cdot y) \leq x^2 + y^2$ for each $x, y \in \mathbf{R}$.

Proof of question 1.b:

From part (a), we know that $z^2 \geq 0 \ \forall \ z \in \mathbf{R}$. Let $z = x + (-y)$

$\Rightarrow (x+(-y))^2 \geq 0 \Rightarrow (x+(-y))(x+(-y)) \geq 0$	Definition of ^{2}.
$\Rightarrow x(x+(-y)) + (-y)(x+(-y)) \geq 0$	Axiom P9
$\Rightarrow xx + x(-y) + (-y)x + (-y)(-y) \geq 0$	Axiom P9
$\Rightarrow x^2 + x(-y) + x(-y) + yy \geq 0$	Proposition 1.1.1.c and Axiom P5
$\Rightarrow x^2 + 1x(-y) + 1x(-y) + y^2 \geq 0$	Axioms P7 and P5
$\Rightarrow x^2 + y^2 + (1+1)x(-y) \geq 0$	Axioms P1, P5, and P9
$\Rightarrow x^2 + y^2 + 2x(-y) \geq 0$	Definition of 2
$\Rightarrow x^2 + y^2 + 2x(-y) + -(2x(-y)) \geq 0 + (-(2x(-y)))$	Axiom O4
$\Rightarrow x^2 + y^2 + 0 \geq -(2x(-y))$	Axiom P3

Claim 2: $-(2x(-y)) = 2xy$

Proof of claim 2:

$2x(-y)+(-(2x(-y)))=0$	Axiom P4
$2x(-y)+2xy=2x(y+(-y))$	Axiom P9
$2x(y+(-y))=2x(0)=0$	Axiom P4 and claim 1
$\Rightarrow 2x(-y)+(-(2x(-y)))=2x(-y)+2xy$	Transitivity
$\Rightarrow -2x(-y)=2xy$	Proposition 1.1.1.a
Thus, $x^2+y^2 \geq 2xy \Leftrightarrow 2\cdot(x\cdot y) \leq x^2+y^2$	Q.E.D.