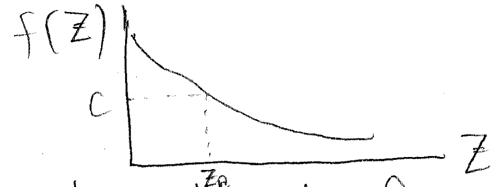
Organizations and the Market



Garicano on hierarchies

$\Omega \subset \mathbb{R}^+$: set of potential problems

$Z \in \Omega$, distr $F(Z)$, density $f(Z)$ • will rank problems by frequency

$A \subset \Omega$: knowledge set of an individual

t_p: time spent in production

Expected output of a single individual: $E[x] = t_p \int_A dF(Z)$

Cost of learning A is $C(A) = c\mu(A)$, where $\mu(A)$ is size of interval

- e.g. let $A = [0, Z]$, then cost is $c(Z - 0) = cZ$.

Expected net output of an individual per unit of time is

- $E[y] = Pr[Z \leq Z_a] - cZ_a$
  $= \int_0^{Z_a} f(\varphi)d\varphi - cZ_a \Rightarrow$ choose $c = f(Z_a^*)$

What if we have people work as a team?

- t^h: time spent "helping," helping cost h - spent whether or not know the solution

Organization: a partition of workers into L classes of size β_i (with $\sum_{i=1} \beta_i = 1$), and to each class is associated:

(i) $A_i \subset \Omega$

(ii) classes l_i from which you can ask for solutions

(iii) t_i^h - time spent helping, t_i^P - time spent producing
$(t_i^h + t_i^P \leq 1 \ \forall i)$

Note: $l <_k i$ means that l precedes i in list of class k

- will ask class l before asking class i.

Time spent helping by class i: $\underbrace{\beta_i t_i^h}_{=T_i^h} = \sum_{k: i \in l_k} \underbrace{\beta_k t_k^p}_{\equiv T_k^p \text{ problems that arise}} \underbrace{[1 - F[\bigcup_{l <_k i} A_l]]}_{\text{that have not already been solved}} \underbrace{h}_{\text{time cost of solving}}$

- all the demands originate in production

Net output: $y = \sum_{i=1}^{L} [\beta_i t_i^p F[\bigcup_{k \in l_i} A_k] - c\beta_i \mu(A_i)]$ crs wrt β_i

Problem: Choose $\beta_i, A_i, l_i, t_i^p, t_i^h$ s.t. $\sum_{i=1}^{L} \beta_i = 1$, $t_i^h + t_i^p \leq 1$

Prop 1: $\forall$ allocations of knowledge, one class specializes in production. The others in problem solving.

Proof: - Time spent helping: $T_i^h = \beta_i t_i^h = \sum_{k: i \in l_k} \beta_k t_k^p (1 - F(\bigcup_{l <_k i} A_l)) h$

- Optimum: $t_i^h + t_i^p = 1 \;\forall i \Rightarrow T_i^h = \beta_i - T_i^p$

$$\Rightarrow \beta_i = T_i^p + \sum_{k: i \in l_k} T_k^p [1 - F(\bigcup_{l <_k i} A_l)] h \quad \text{for } i = 1, \dots, L$$

- given A_i and β_i, have L equations in L unknowns T_i^p

$$\Rightarrow T_i^p = \rho_{i1}\beta_1 + \dots + \rho_{iL}\beta_L = \rho_i' \beta = [\rho_{i1} \cdots \rho_{iL}] \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_L \end{bmatrix}$$

- Net output becomes

$$y = \sum_{i=1}^{L} [\underbrace{T_i^p}_{\rho_i'\beta} F[\bigcup_{k \in l_i} A_k] - c\beta_i \mu(A_i)]$$

max y s.t. $\sum_{i=1}^{L} \beta_i = 1$, $T_i^P \in [0, \beta_i]$

i.e. $\beta_i \geq \beta_i' \beta \geq 0$

Thus, either $T_i^P = 0$ or $T_i^P = \beta_i$

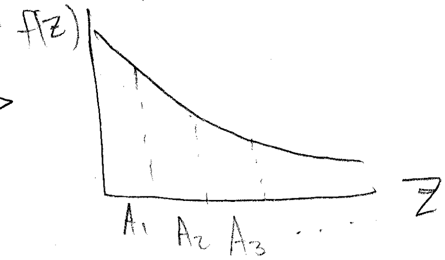
Prop 2: Knowledge sets have empty intersections.

Prop 3: Production workers learn the most common problems and "higher up" in the list, the less common problems one learns. "Management by exception"

Proof: Class i spends time helping production workers w in amount $[1 - F[\bigcup_{l<w_i} A_l]]h \frac{\beta_w}{\beta_i}$

assume w knows $[Z_w, Z_w']$ but not $\underbrace{[0, \varepsilon]}_{\text{known by class } i}$. Then, can swap $[Z_w' - \varepsilon, Z_w']$ and $[0, \varepsilon]$ in the knowledge set.

$\Rightarrow$



Prop 4: The organization is pyramidal in that each layer has a smaller size than the "previous" one.

$\beta_i = [1 - F(Z_{i-1})]h\beta_0 \qquad [0, Z_i] = \bigcup_{j<i} A_j$

$\beta_{i+1} = [1 - F(Z_i)]h\beta_0$