

5/2/08

14.129

1

The easiest way to do bargaining under incomplete information is a TIOLO by the uninformed party.

Cramer, Pratt, Goricano

Language is a way of coordinating or communication

This is a paper about bounded rationality - people can only communicate so much

• Team theoretic

• Engineer-salesman

◦ clients with events "that demand solutions."

◦ events: $x \in X$ (finite)

◦ prob $f_x > 0$

• Bounded rationality: language/code C is a partition of X . e.g. $X = \{x_1, x_2, x_3\}$, and language has at most 2 words. Each element of the partition is a word.

$$\{C\} = \left\{ \left\{ \underbrace{\{x_1, x_2\}}_{w_1}, \underbrace{\{x_3\}}_{w_2} \right\}, \left\{ \underbrace{\{x_1\}}_{w_1}, \underbrace{\{x_2, x_3\}}_{w_2} \right\} \right\}$$

more generally

$C = \{w_1, \dots, w_k\}$, breadth of word k_k is the number of elements in w_k . The frequency of word k is

$$p_k = \sum_{j \in w_k} f_j$$

• Engineer needs to diagnose the problem. The cost of doing so is increasing in n_k . $d(n_k) \uparrow$ in n_k

• Expected diagnosis cost $D(C, f) = \sum_{k=1}^K p_k d(n_k)$

◦ There is no notion of "closeness" of words.

• For a given K , what is the best partition C ?

5/2/08

14.129

2

Prop: In an optimal code, $n_k \geq n_{k'} \Rightarrow f_x \leq f_{x'}$ for all $x \in W_k, x' \in W_{k'}$.

◦ less precise words must relate to less frequent events.

◦ If not, switch two events, and gain is

$$[d(n_k) - d(n_{k'})][f_x - f_{x'}] > 0$$

◦ frequent events must be identified more precisely

Prop: If d is convex in n , (ie $d(2) - d(1) > d(1) - d(0)$)

then $n_k - n_{k'} \geq 2 \Rightarrow p_{k'} \geq p_k$

◦ precise words should be more frequent.

◦ If not, switch one event and $D(c) - D(\bar{c})$

$$= d(n_k)p_k + d(n_{k'})p_{k'} - d(n_k - 1)(p_k - f_j) - d(n_{k'} + 1)(p_{k'} + f_j)$$

$$\text{where } j \in W_k. \leq d(n_k) - d(n_k - 1)$$

$$= [d(n_k) - d(n_k - 1)]p_k - [d(n_{k'} + 1) - d(n_{k'})]p_{k'}$$

$$+ f_j [d(n_k - 1) - d(n_{k'} + 1)]$$

$$> \underbrace{[d(n_k) - d(n_k - 1)]}_{\geq 0} \underbrace{(p_k - p_{k'})}_{\leq 0} \geq 0 \text{ by convexity}$$

P.g:

Suppose $X = \{x_1, x_2, x_3\}$, $f(x_1) = f(x_2) = \frac{1-p}{2}$, $f(x_3) = p > \frac{1}{3}$

then, optimal $C = \{\{x_1, x_2\}, \{x_3\}\}$.

$$D(C) = p + 2(1-p) \text{ if } d(1) = 1, d(2) = 2$$

$$= 2 - p$$

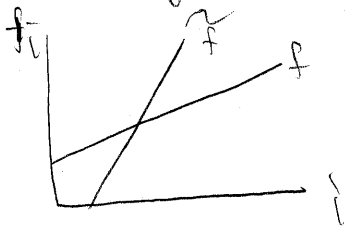
◦ environment is more complex if p is smaller

Rank events s.t. $f_1 \leq f_2 \leq \dots$

• $F_i \equiv \Pr [j \leq i] = \sum_{j=1}^i f_j$

Definition: \tilde{f} is less complex than f if $F_i \geq \tilde{F}_i \forall i$,

where \tilde{F}_i uses the "indexing" of F_i .

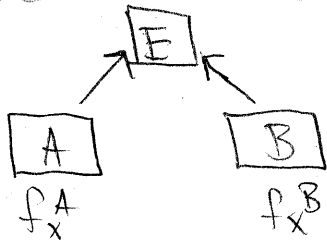


\tilde{f} less complex than f

Proposition: f more complex than $\tilde{f} \Rightarrow$ best code under f will be more costly than the best code under \tilde{f} : $\min DCC(f) \geq \min DCC(\tilde{f})$

Moreover, an increase in the number of words from 1 to $K > 1$ lowers C more for less complex environments but going to very large numbers of words decreases C more for more complex environments

What if we have one engineer and two salesmen?



• should A and E have "different dialects" than B and E?

Prop: Optimally, $C_A = C_B$

Pf: Take two codes C_A, C_B s.t. $C_A \cup C_B$ contains K words.

- If $C_A \neq C_B$, then both have at most $K-1$ words
- Take narrowest noncommon word. If it belongs to smallest n_k

C_A , add it to C_B , while taking the underlying

5/2/08

14.129

4

events away from C_B words.

eg. $C_A = \{ \{1,4\}, \{2,5\}, \{3,6\} \}$

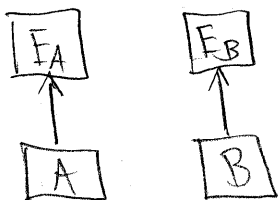
$C_B = \{ \{1,2,3\}, \{4,5,6\} \}$

$\Rightarrow C_{B'} = \{ \{1,4\}, \{2,3\}, \{5,6\} \} \Rightarrow C_{B''} = \{ \{1,4\}, \{2,5\}, \{3\}, \{6\} \}$

Then, $\forall x$ length of word in new $C_{B'}$ containing x is not larger than in C_B . Moreover, $C_{B'}$ has at most K words.

Some x 's are in strictly narrower words.

What about this structure?



why work together?

Integration makes more sense with more homogeneity in underlying populations.

o pooling can help deal with more customers.