Stage 1: (i) 1 announces θ^1

(ii) 2 agrees ($\to$ stage 2) or challenges: $\tilde{\theta}^1 \neq \theta^1$

(iii) if challenge, 1 chooses between:

$\{x, -t_x - \Delta t, t_x - \Delta t\}$ or $\{z, -t_z - \Delta t, t_z + \Delta t\}$

s.t. $\theta^1 x - t_x > \theta^1 z - t_z \Rightarrow \theta^1(x-z) > t_x - t_z$

$\tilde{\theta}^1 x - t_x < \tilde{\theta}^1 z - t_z \quad \tilde{\theta}^1(x-z) < t_x - t_z$

$\tilde{\theta}^1(x-z) < t_x - t_z < \theta^1(x-z)$

Stage 2: Same, with roles reversed

if $x > z \Rightarrow \tilde{\theta}^1 < \frac{t_x - t_z}{x-z} < \theta^1$

If no, implement $y(\theta), t^1(\theta), t^2(\theta)$

if $x < z \Rightarrow \tilde{\theta}^1 > \frac{t_x - t_z}{x-z} > \theta^1$

- $d \in \{0,1\}$
- $u^1 = \theta^1 d + t^1$
- $u^2 = \theta^2 d + t^2$

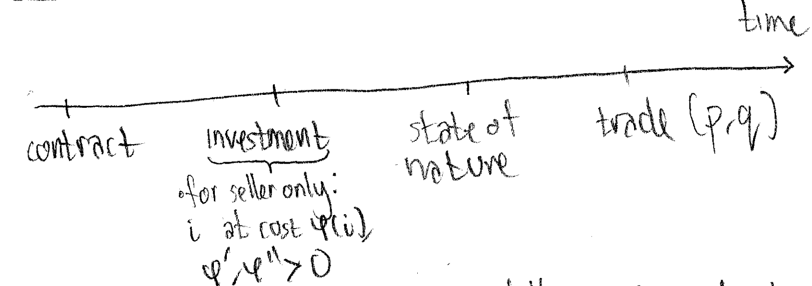
- e.g. if $\theta^1 > \tilde{\theta}^1$, choose $x=1, z=0$
  $\theta^1 < \tilde{\theta}_1$, choose $x=0, z=1$

Monotonicity is violated here: For example:

- efficiency: $y(\theta) = 1$ iff $\theta^1 + \theta^2 \geq 0$
- $t^1(\theta) = -t^2(\theta)$
  $= \dfrac{\theta^2 - \theta^1}{2}$
- take θ s.t. $\theta^1 + \theta^2 \geq 0$
- take $\hat{\theta}$ s.t. $\hat{\theta}^2 = \theta^2$ and $\hat{\theta}^1 > \theta^1$
  $\Rightarrow$ want to redistribute more, but original allocation is still a NE.

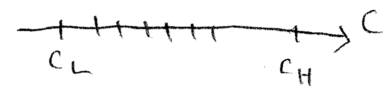
With two people, not renegotiation-proof.
Any deviation from rationality can break this mechanism.
These arbitrarily large penalties underly a lot of such mechanisms. This is abusing sequential rationality.

Hart-Moore, Segal



Goal: Trade a "widget": only 1 type is valuable or "special." It generates value $v > c$. $c \in \{c_L, c_H\}$. $c_L < c_H < v$.

- $\Pr[c = c_L] = i$
- the "type" of widget is not verifiable.
- there exist generic widgets, N-1 of them, with costs c_n^g, $n = 1, \ldots, N-1$. $c_n^g = c_L + \frac{n}{N}(c_H - c_L)$



  - those have zero value
  - problem is more complex if N is large.

State of nature: permutation of the N widgets, and a $c \in \{c_L, c_H\}$. Assume all permutations are all equally likely.

First best: always trade. always trade valuable widget:

$$\max_i \; i(v - c_L) + (1-i)(v - c_H) - \varphi(i)$$

$$(i): \quad \varphi'(i) = c_H - c_L$$

No renegotiation: First best can be achieved, even by a contract not specifying any other outcome than no trade

- seller can make a TIOLI offer to buyer

Renegotiation: (w/result being p_L after c_L and p_H after c_H)

- seller: $\max_i \; i(p_L - c_L) + (1-i)(p_H - c_H) - \varphi(i)$

FB requires $p_L = p_H$

- assume full bargaining power for buyer $\Rightarrow p_j = c_j \Rightarrow i = 0$

State of Nature: $(\underbrace{L}_{\text{special widget costs } c_L}, \underbrace{\tau}_{\text{a permutation of the N widgets}})$

Widgets	1	2	3 ...	N-2	N-1	N
Costs in (L,τ)	$\underbrace{c_L}_{\text{special widget}}$	$c_L+\frac{1}{N}\Delta c$	...		$c_L+\frac{N-2}{N}\Delta c$	$c_L+\frac{N-1}{N}\Delta c$
Costs in (H,τ')	$c_L+\frac{1}{N}\Delta c$	$c_L+\frac{2}{N}\Delta c$	...		$c_L+\frac{N-1}{N}\Delta c$	$\underbrace{c_H}_{\text{special}}$

- $\Delta c = c_H - c_L$

- Mechanism M, w/equilibrium strategies
  - $m^B(L,\tau)$, $m^S(L,\tau)$ $\mapsto$ price $p(L,\tau)$

    $\mapsto$ surplus, since renegotiation implies ex-post efficiency,

    $\begin{cases} v - p(L,\tau) \\ p(L,\tau) - c_L \end{cases}$

- In state (H,τ'), $m^B(H,\tau')$, $m^S(H,\tau') \mapsto p(H,\tau')$
  - surplus: $\begin{cases} v - p(H,\tau') \\ p(H,\tau') - c_H \end{cases}$
- want incentive compatibility: in (H,τ'), seller should not want to play $m^S(L,\tau)$. In (L,τ), buyer should not want to play $m^B(H,\tau')$
  - thanks to renegotiation, this is a zero sum game after investments are sunk
  - in the limit, will have no benefit of contracting