New Keynesian model

Price staggering leads to price rigidity.

aside: Money is neutral if a one-time, unexpected permanent change has a one-for-one change in the price level, and no other changes.

Staggering generates price stickiness, but not inflation stickiness.

alternative structures:

- vertical chains of production
- this can lead to amplification
- what happens if each price setter follows a state-dependent rule? (instead of time dependent as above.)
- there will be no rigidity in the aggregate in this framework
- much less stickiness, since the prices that are most far out of whack are the ones most likely to change
- learning leads to some stickiness
- Rational inattention. Collecting information is costly

We don't know how robust inflation stickiness is. We do think it exists in the data. (inflation inertia)

- We don't have a good theory for why this might hold.

What about wage staggering instead of price staggering?

- 1/2 of wages are set in Jan for a year and 1/2 in Jun for a year
- $w_t = 0.5(p_t + E_t[p_{t+1}]) + 0.5a(x_t + E_t[x_{t+1}])$
- $p_t = 0.5(w_t + w_{t-1})$

In terms of relative wages:

$$w_t = 0.5(w_{t-1} + E_t[w_{t+1}]) + a(x_t + E_t[x_{t+1}])$$

What is the importance here of wage stickiness versus price stickiness?

- wages are typically set for a year.

How do we think about sales and their implications for price stickiness?

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The "New Keynesian" model:

- consumption/saving choice ⇒ role of interest rate
- leisure/work choice ⇒ employment fluctuations
- money/bond choice ⇒ determination of interest rate
- nominal rigidities ⇒ choose Calvo specification
- continuum of households

$$\max E\left[\sum_{k=0}^{\infty} \beta^k \left[U(C_{i,t+k}) + \underbrace{V\left(\frac{M_{i,t+k+1}}{P_{t+k}}\right)}_{\text{depends on real money balances}} - Q(N_{i,t+k})\right] \middle| \Omega_t\right]$$

This is the consumption side

subject to:

$$C_{it} \equiv \left[ \int_0^1 C_{i,j,t}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \qquad \bar{P}_t = \left[ \int_0^1 P_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}$$

$$\int_0^1 P_{jt} C_{ijt}\, dj + M_{i,t+1} + B_{i,t+1} = W_t N_{it} + (1+i_t)\underbrace{B_{it}}_{\text{bonds have positive return}} + M_{it} + \Pi_{it} + \underbrace{X_{it}}_{\text{gov't transfers}}$$

- No capital accumulation equation

FOCs are:

- $C_{i,j,t} = \left(\frac{P_{jt}}{\bar{P}_t}\right)^{-\sigma} C_{it}$

- $u'(C_{it}) = E\left[\beta(1+r_{t+1})u'(C_{i,t+1}) \mid \Omega_t\right]$
- $V'\left(\frac{M_{i,t+1}}{\bar{P}_t}\right) / u'(C_{i,t}) = \frac{i_{t+1}}{1+i_{t+1}}$ opportunity cost of holding money

- $\frac{W_t}{\bar{P}_t} u'(C_{i,t}) = Q'(N_{i,t})$ labor-leisure choice

## Production side

Calvo price-setters
- prob $\delta$ can set prices at time $t$

- $Y_{jt} = Z_t N_{jt}$

$$\max E\left[ \sum_{k=0}^{\infty} \beta^k \frac{u'(C_{t+k})}{u'(C_t)} (1-\delta)^k \left( \frac{P_{jt}}{\bar{P}_{t+k}} Y_{j,t+k} - \frac{W_{t+k}}{\bar{P}_{t+k}} \overbrace{\frac{Y_{j,t+k}}{Z_{t+k}}}^{N_{j,t+k}} \right) \Bigg| \Omega_t \right]$$

$$\text{s.t.} \quad Y_{j,t+k} = \left(\frac{P_{jt}}{P_{t+k}}\right)^{-\sigma} \underbrace{Y_{t+k}}_{\text{aggregate output}} \quad \text{(demand)} \quad \text{(in eq., will be } C_{t+k})$$

FOC:

$$P_{jt} = P_t = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{mark-up}} \frac{\overbrace{E_t\left[\sum_{k=0}^{\infty} A(k)(W_{t+k}/Z_{t+k}) \mid \Omega_t\right]}^{\text{average marginal cost}}}{E_t\left[\sum_{k=0}^{\infty} A(k) \mid \Omega_t\right]}$$

where $A(k) = \beta^k \frac{U'(C_{t+k})}{U'(C_t)} (1-\delta)^k (\bar{P}_{t+k})^{\sigma-1} Y_{t+k}$

Price level is:

$$\bar{P}_t = \left[(1-\delta)\bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

General equilibrium

In equilibrium, will have:

- $C_{it} = C_t = Y_t$
- $N_t = Y_t / Z_t$
- $M_{it} = M_t$

The FOCs become: (demand)

(IS): $U'(Y_t) = E_t[\beta(1+r_{t+1})U'(Y_{t+1}) \mid \Omega_t]$

(LM): $V'\left(\frac{M_{t+1}}{\bar{P}_t}\right) / U'(Y_t) = \frac{i_{t+1}}{1+i_{t+1}}$

The others are: (supply)

(LS): $\frac{W_t}{\bar{P}_t} U'(Y_t) = Q'\left(\frac{Y_t}{Z_t}\right)$

(PS): $P_t = \frac{\sigma}{\sigma-1} \frac{E[\sum_k A(k)(W_{t+k}/Z_{t+k}) \mid \Omega_t]}{E[\sum_k A(k) \mid \Omega_t]}$

where $A(k) = \beta^k \dfrac{u'(\bar{Y}_{t+k})}{u'(\bar{Y}_t)} (1-\delta)^k (\bar{P}_{t+k})^{\sigma-1} Y_{t+k}$

$$\bar{P}_t = \left[(1-\delta)\bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

(PF): $N_t = Y_t / Z_t$

## Second-best

Without nominal rigidities, (LS) becomes

$$\frac{W_t}{\bar{P}_t} = \frac{Q'(Y_t/Z_t)}{U'(Y_t)}$$

(PS) gives $\bar{P}_t = \dfrac{\sigma}{\sigma-1}\dfrac{W_t}{Z_t} \Leftrightarrow \dfrac{W_t}{\bar{P}_t} = \dfrac{\sigma-1}{\sigma} Z_t$

$$\Rightarrow \frac{Q'(Y_t/Z_t)}{u'(Y_t)} = \frac{\sigma-1}{\sigma} Z_t$$

If utility is logarithmic, this becomes:

$$Q'\left(\frac{Y_t}{Z_t}\right)\frac{Y_t}{Z_t} = \frac{\sigma-1}{\sigma}$$

- uniquely determines $Y_t/Z_t$. $(= N_t)$
- employment is invariant to shocks.
  - this is a result of the log utility assumption.

With no capital accumulation, consumption moves one-for-one with wage.