Blanchard-Fischer pp264-266

$X_t = a X_{t-1} + b E_t[X_{t+1}] + Z_t$

Lag operator: $L E_t[X_t] \equiv E_t[X_{t-1}] = X_{t-1}$

$L^{-1} E_t[X_t] \equiv E_t[X_{t+1}]$

$\Rightarrow E_t[X_t] = a L X_t + b L^{-1} E_t[X_t] + E_t[Z_t]$

$\Rightarrow \left(\frac{1}{bL} - \frac{a}{b} - \frac{1}{L^2}\right) L E_t[X_t] = \frac{E_t[Z_t]}{b}$

$\Rightarrow \left(L^{-2} - \frac{1}{b} L^{-1} + \frac{a}{b}\right) L E_t[X_t] = -\frac{E_t[Z_t]}{b}$

- $\lambda_1, \lambda_2 \Rightarrow \lambda_1 + \lambda_2 = \frac{1}{b}$

$\lambda_1 \lambda_2 = \frac{a}{b}$

Unique solution: Want $|\lambda_1| < 1$, $|\lambda_2| > 1$

Then $(L^{-1} - \lambda_1)(L^{-1} - \lambda_2) L E_t[X_t] = -\frac{E_t[Z_t]}{b}$

$L^{-1} - \lambda_2 = \frac{1}{L} - \lambda_2 = \frac{1 - \lambda_2 L}{L}$

$\Rightarrow \frac{(1 - \lambda_1 L)}{\cancel{L}} \left(\frac{1 - L\lambda_2}{L\lambda_2}\right) \cancel{L} E_t[X_t] = -\frac{E_t[Z_t]}{b\lambda_2}$

$\Rightarrow (1 - (\lambda_2 L)^{-1})(1 - \lambda_1 L) E_t[X_t] = \frac{E_t[Z_t]}{b\lambda_2}$

$$\Rightarrow (1-\lambda_1 L) E_t[X_t] = \frac{1}{b\lambda_2} \sum_{i=0}^{\infty} (\lambda_2 L)^{-i} E_t[Z_t]$$

$$= \frac{1}{b\lambda_2} \sum_{i=0}^{\infty} \lambda_2^{-i} E_t[Z_{t+i}]$$

$$= \frac{1}{b\lambda_2} Z_t \quad \text{if } Z_t \overset{iid}{\sim} \underbrace{D}_{\text{arbitrary distribution}}(0,\sigma)$$

$$\Rightarrow X_t = \lambda_1 X_{t-1} + \frac{1}{b\lambda_2} \sum_{i=0}^{\infty} \lambda_2^{-i} E_t[Z_{t-i}] + c\lambda_1^t + d\lambda_2^t$$

° general solution

We know that $(1-\lambda L) c\lambda^t = c\lambda^t - c\lambda\lambda^{t-1} = c\lambda^t - c\lambda^t = 0$

Lecture notes 7, page 13:

$$\circ\, p_t = b p_{t-1} + b E_t[p_{t+1}] + (1-2b) E_t[m_t]$$

$$\Rightarrow p_t (1 - bL - bL^{-1}) = (1-2b) E_t[m_t]$$

$$\Rightarrow \left(\frac{1}{bL} - 1 - \frac{1}{L^2}\right) L p_t = \frac{(1-2b)}{b} E_t[m_t]$$

$$\Rightarrow \underbrace{\left(L^{-2} - \frac{1}{b} L^{-1} + 1\right)}_{\Rightarrow \lambda_1 \lambda_2 = 1 \Rightarrow \lambda_1 = 1/\lambda_2 \Rightarrow \lambda_2 > 1} L p_t = -\left(\frac{1-2b}{b}\right) E_t[m_t]$$

$$\Rightarrow (1-\lambda_1 L)(1-(\lambda_2 L)^{-1}) p_t = -\left(\frac{2b-1}{b\lambda_2}\right) E_t[m_t]$$

$$\Rightarrow (1-\lambda_1 L)p_t = -\left(\frac{2b-1}{b\lambda_2}\right)\sum_{i=0}^{\infty}(\lambda_2 L)^{-i} E_t[m_t]$$

$$= \left(\frac{1-2b}{b\lambda_2}\right)\sum_{i=0}^{\infty}\lambda_1^i \, E_t[m_{t+i}]$$

since $\dfrac{1-2b}{b\lambda_2} = (1-\lambda_1)^2 \qquad (*)$

$$\Rightarrow p_t = \lambda_1 p_{t-1} + (1-\lambda_1)^2 \sum_{i=0}^{\infty}\lambda_1^i E_t[m_{t+i}]$$

allows us to write prices as functions of past prices and future expectations.

To show (*),

$(1-\lambda_1)^2 = 1+\lambda_1^2 - 2\lambda_1$, where

$$\lambda_1 = \frac{\frac{1}{b} \pm \sqrt{\frac{1}{b^2}-4}}{2} = \frac{\frac{1}{b} \pm \sqrt{\frac{1-4b^2}{b^2}}}{2} = \frac{1 \pm \sqrt{1-4b^2}}{2b} = \frac{1-\sqrt{1-4b^2}}{2b}.$$

$$= 1 + \left(\frac{1}{4b^2} + \frac{1-4b^2}{4b^2} - 2\,\frac{1}{2b}\,\frac{(1-4b^2)^{1/2}}{2b}\right) - 2\left[\frac{1}{2b} - \frac{(1-4b^2)^{1/2}}{2b}\right]$$

$$= 1 + \frac{1}{2b^2} - 1 - \frac{1}{2b^2}(1-4b^2)^{1/2} - \frac{1}{b} + \frac{(1-4b^2)^{1/2}}{b}$$

$$= \frac{1}{2b^2} - \frac{1}{b} + (1-4b^2)^{1/2}\left(\frac{1}{b} - \frac{1}{2b^2}\right)$$

$$= \frac{1}{2b^2}\left[(1-2b)\left(1-(1-4b^2)^{1/2}\right)\right] = \frac{1-2b}{b}\left[\underbrace{\frac{1-(1-4b^2)^{1/2}}{2b}}_{=\lambda_1}\right]$$

$$= \frac{1-2b}{b\lambda_2}$$

Sticky Price Model (Calvo)

- Firms desired price: (all in logs)

$$p_t^* = p_t + \alpha \underbrace{y_t}_{\text{log of output gap}} \qquad (1)$$

- Boom $\Rightarrow$ demand $\uparrow$ $\Rightarrow$ mc $\uparrow$ $\Rightarrow$ $p^* \uparrow$
- Price adjustment
  - λ firms adjust every period
  - $\underbrace{X_t}_{\text{desired price}} = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i E_t[p_{t+i}] \qquad (2)$

Overall price level

- $p_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i X_{t-i}$

Phillips Curve: relates π and y

$$(2) \Rightarrow X_t = \lambda E_t[p_t^*] + \lambda(1-\lambda) \sum_{j=0}^{\infty} (1-\lambda)^j E_t[p^*_{t+1+j}]$$

$$\Rightarrow X_t = \lambda p_t^* + (1-\lambda) E_t[X_{t+1}]$$

Will give us: $p_t = \lambda X_t + (1-\lambda) p_{t-1}$

or $X_t = \frac{p_t}{\lambda} - \frac{1-\lambda}{\lambda} p_{t-1}$

Skipping some steps,

$$\frac{p_t}{\lambda} - \frac{(1-\lambda)}{\lambda} p_{t-1} = \lambda p_t^* + (1-\lambda) E_t\left[\frac{p_{t+1}}{\lambda} - \frac{1-\lambda}{\lambda} p_t\right]$$

Define $\pi_t = p_t - p_{t-1}$

$$\Rightarrow \pi_t = \frac{\lambda^2 \alpha}{1-\lambda} y_t + E_t[\pi_{t+1}]$$

using the method earlier,

$$\pi_t (1 - L^{-1}) = \frac{\lambda^2 \alpha}{1-\lambda} y_t$$

$$\Rightarrow \pi_t = \frac{\lambda^2 \alpha}{1-\lambda} \sum_{i=0}^{\infty} E_t[y_{t+i}]$$

- inflation depends on expected future output gaps.

- demand: $m_t = p_t + y_t$

Fischer model:

- $\frac{1}{2}$ of the firms change price each period (they alternate)

- $p_{1t} = E_{t-1}[\underbrace{p_{1t}^*}_{\text{desired price}}]$

- $p_{2t} = E_{t-2}[p_{2t}^*]$

- $p_t^* = p_t + \alpha y_t = p_t + \alpha(m_t - p_t) = (1-\alpha)p_t + \underbrace{\alpha\, m_t}_{\text{weight given to macro vars.}}$

- $p_t = \frac{1}{2}(p_{1t} + p_{2t})$

$p_{1t} = f(p_{2t}, m_t)$

$$p_{1t}^* = (1-\alpha)p_t + \alpha m_t$$
$$= (1-\alpha)\left[\tfrac{1}{2}(p_{1t} + p_{2t})\right] + \alpha m_t$$

- know $p_{1t} = E_{t-1}[p_{1t}^*]$

- $p_{1t} = (1-\alpha)\frac{1}{2}(p_{1t}+p_{2t}) + \alpha E_{t-1}[m_t]$
- $p_{2t} = E_{t-2}[p_{2t}^*]$

$$\Rightarrow p_{1t} = \frac{2\alpha}{1+\alpha} E_{t-1}[m_t] + \frac{1-\alpha}{1+\alpha} p_{2t} \qquad (1)$$

Similarly, get

$$p_{2t} = \frac{2\alpha}{1+\alpha} E_{t-2}[m_t] + \frac{1-\alpha}{1+\alpha} \underbrace{E_{t-2}[p_{1t}]} \qquad (2)$$

to replace this, need to take expectations of (1). (E_{t-2})

$$E_{t-2}[p_{1t}] = E_{t-2}[m_t]\frac{2\alpha}{1+\alpha} + \frac{1-\alpha}{1+\alpha} p_{2t}$$

$$\Rightarrow p_{2t} = \frac{2\alpha}{1+\alpha} E_{t-2}[m_t] + \frac{1-\alpha}{1+\alpha}\left[\frac{2\alpha}{1+\alpha} E_{t-2}[m_t] + \frac{1-\alpha}{1+\alpha} p_{2t}\right]$$

Simplifying, we get

$p_{2t} = E_{t-2}[m_t]$. Replace this in expression for p_{1t}.

$$p_{1t} = \frac{2\alpha}{1+\alpha} E_{t-1}[m_t] + \frac{1-\alpha}{1+\alpha} E_{t-2}[m_t]$$

$$p_t = \frac{1}{2}[p_{1t}+p_{2t}] = \frac{\alpha}{1+\alpha} E_{t-1}[m_t] + \frac{1}{1+\alpha} E_{t-2}[m_t]$$

$$p_t = E_{t-2}[m_t] + \frac{\alpha}{1+\alpha}\left[\underbrace{E_{t-1}[m_t] - E_{t-2}[m_t]}_{\text{surprise} \equiv v_{2t}}\right]$$

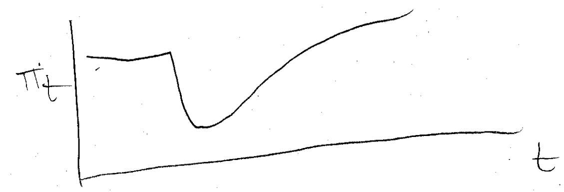
$$\Rightarrow p_t = E_{t-2}[m_t] + \frac{\alpha}{1+\alpha} v_{2t}.$$

$\pi_t = p_t - p_{t-1}$

$p_{t-1} = \frac{\alpha}{1+\alpha} E_{t-2}[m_{t-1}] + \frac{1}{1+\alpha} E_{t-3}[m_{t-1}]$

$$\Rightarrow \pi_t = \frac{\alpha}{1+\alpha}\left[E_{t-1}[m_t] - E_{t-2}[m_{t-1}]\right] + \frac{1}{1+\alpha}\left[E_{t-2}[m_t] - E_{t-3}[m_{t-1}]\right]$$

- inertia in inflation.



Solving for α,

$$y_t = m_t - p_t$$
$$= m_t - E_{t-2}[m_t] - \frac{\alpha}{1+\alpha} u_{2t}$$
$$= \underbrace{m_t - E_{t-1}[m_t]}_{u_{1t}} + \underbrace{E_{t-1}[m_t] - E_{t-2}[m_t]}_{u_{2t}} - \frac{\alpha}{1+\alpha} u_{2t}$$

$$y_t = u_{1t} + \frac{1}{1+\alpha} u_{2t}$$

Mankiw-Reis (2002 QJE) : sticky information transmission