Last time:
- cash in advance
- money in the utility function

These give us a well-defined money demand function.
How does this distort consumption decisions?
- This is like a small tax in the cash in advance model. (Due to opportunity cost of holding money.)

Superneutrality - changes in growth rate don't matter.
Neutrality - changes in levels of money don't matter

These models are useful in hyperinflationary environments.
Why is the rate of money growth less than the inflation rate?
Was hyperinflation a result of money growth?
Did governments maximize seignorage? If not, why not?
What was the role of fiscal policy?

$$\frac{M_t}{P_t} = C_t L(\underbrace{r_t + \pi_t^e}_{\text{nominal interest rate}})$$

demand for money

$L' < 0$

assume $C_t = C$ and $r_t = r$, since these are relatively fixed in the SR.

$$\frac{M_t}{P_t} = C L(r + \pi_t^e)$$
$$= G(\pi_t^e)$$
$\Rightarrow$ just a function of expected inflation.

assume $\frac{M_t}{P_t} = \exp\{-\alpha \pi_t^e\}$

$\Rightarrow m_t - p_t = -\alpha \pi_t^e$, where $m_t = \ln M_t$, $p_t = \ln P_t$

- decreasing in π_t^e

Take time derivatives:

$x_t - \pi_t = -\alpha \frac{d\pi_t^e}{dt}$ $\qquad x_t = \frac{d}{dt} m_t = \frac{\dot{M}_t}{M_t}$

$\pi_t = \frac{\dot{P}_t}{P_t}$

How does π_t^e move?

- assume adaptive expectations.
  - $\frac{d\pi_t^e}{dt} = \beta(\pi_t - \pi_t^e)$

Rearranging: $\frac{d\pi_t^e}{dt} = \frac{\beta}{1-\alpha\beta}(x_t - \pi_t^e)$

(bubble) • if $\alpha\beta > 1$, then unstable. Why? $x_t \rightarrow \pi_t \Rightarrow \frac{d\pi_t^e}{dt} \rightarrow \pi_t \rightarrow \ldots$

(money growth) • if $\alpha\beta < 1$, then equilibrium is stable. Take $x_t > 0$, $\pi_t = 0$.

This converges to $\pi_t = \pi_t^e = x_t$

Cagan estimates α and β. Finds $\alpha\beta < 1$ • if $x\uparrow$, jump in π to above x. Then back to new x.

Seignorage

Maximomum revenue gov't can get from money creation.

$$S_t = \frac{dM_t/dt}{P_t} = \frac{dM_t/dt}{M_t} \frac{M_t}{P_t} = x_t \exp\{-\alpha \pi_t^e\}$$

Choose x_t to maximize (steady state)

- $S = x_t \exp\{-\alpha x_t\}$

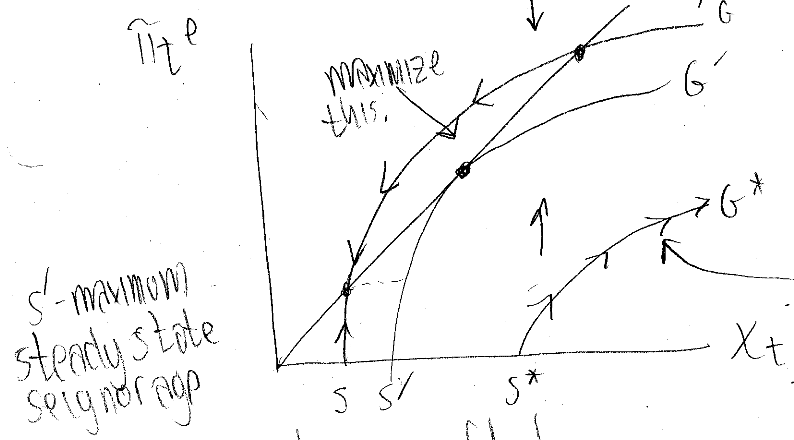
Solution: $x_t^* = \frac{1}{\alpha}$

- This leads to a much smaller rate of money growth

Start from $S_t = x_t \exp\{-\alpha \pi_t^e\}$ (outside steady state)

$$\Rightarrow \pi_t^e = \frac{1}{\alpha} \ln\left(\frac{x_t}{S_t}\right)$$

- concave in π_t^e, x_t space,



- which of these is stable?

$$\frac{d\pi_t^e}{dt} = \frac{\beta}{1-\alpha\beta}(x_t - \pi_t^e)$$

need to stay on the locus to maintain the same amount of seignorage.

- under hyperinflation, you want G* path
  - will lead to increasing rates of inflation.

<u>Nominal rigidities</u>

- $\frac{M_t}{P_t} = C_t L(r_t + \pi_t^e)$
  - gives a relation between P_t and π_t^e.
- $m_t - p_t = -\alpha(\underbrace{E[p_{t+1} | \Omega_t] - p_t}_{\pi_{t+1}^e})$ in discrete time
  - gives us p_t as a function of future expectations.

- $P_t = \frac{1}{1+\alpha}\left(\sum_{i=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^i E[M_{t+i} | \Omega_t]\right)$

The price level is not an asset price. It is an average of millions of prices. It will not move as described above.

If P_t adjusts more slowly, what will happen?
If P_t doesn't adjust quickly to changes in M_t, then the adjustment will have to come from i_t.
- Now, we have non-neutrality.
- increase in M_t will lead to decrease in i_t and likely a decrease in r_t
  - then it can affect C_t: Here, it would be higher.

What happens to supply? If firms have monopoly power, they may increase quantity if prices are fixed.

3 steps:
1] Static model. Monopolistic competition, price setting
2] More realistic price staggering rules.
3] Input all this into an RBC model with price setting.