# Introducing Money

Cash-in-advance model:

Ignore uncertainty, labor/leisure choice

$$\max \sum_{i=0}^{\infty} \beta^i U(C_{t+i})$$

$$(\lambda_t) \quad \text{s.t. } P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1+i_t) B_t + (1+r_t) P_t K_t + X_t$$

$$(\mu_t) \quad \text{and} \quad P_t C_t \leq M_t + X_t$$

- $P_t$ - price of goods
- $M_t$ - money
- $B_t$ - bonds
- $K_t$ - capital

  } at start of $t$ (for $M_t$, $B_t$, $K_t$)
- $\Pi_t / W_t$ - profits/wages
- $r_t$ - rental rate
- $i_t$ - nominal interest rate
- $X_t$ - nominal transfer from government

Clearly, the $P_t C_t \leq M_t + X_t$ constraint will be important in explaining why people hold money.

- $X_t$ is independent of $M_t$. $X_t$ is determined before $M_t$ needs to be decided.

FOCs:

$(C_t)$: $u'(C_t) = (\lambda_t + \mu_t) P_t \quad \Rightarrow \quad \underbrace{\frac{1}{P_t} u'(C_t)}_{\text{marginal utility from a dollar}} = \lambda_t + \mu_t$

$(M_{t+1})$: $\lambda_t = \beta(\lambda_{t+1} + \mu_{t+1})$

$$\left.\begin{aligned} (B_{t+1}) &: \lambda_t = \beta(1+i_{t+1})\lambda_{t+1} \\ (K_{t+1}) &: \lambda_t P_t = \beta(1+r_{t+1})\lambda_{t+1} P_{t+1} \end{aligned}\right\} (*)$$

$(*) \Rightarrow \quad \beta(1+i_{t+1}) = \beta(1+r_{t+1}) \frac{P_{t+1}}{P_t}$

$$\frac{P_{t+1}}{P_t} = \frac{P_t}{P_t} + \underbrace{\frac{P_{t+1}-P_t}{P_t}}_{\equiv \pi_{t+1}} \approx 1 + \pi_{t+1}$$

$\Rightarrow \quad (1+i_{t+1}) = (1+r_{t+1})(1+\pi_{t+1})$

First two:

$$\lambda_t = \beta \frac{u'(C_{t+1})}{P_{t+1}}, \quad \lambda_{t+1} = \beta \frac{u'(C_{t+2})}{P_{t+2}}$$

$\Rightarrow$ From $(B_{t+1})$:

$$\beta \frac{u'(C_{t+1})}{P_{t+1}} = \beta(1+i_{t+1})\beta \frac{u'(C_{t+2})}{P_{t+2}}$$

$$\Rightarrow \quad \frac{u'(C_{t+1})}{P_{t+1}} = \beta(1+i_{t+1}) \frac{u'(C_{t+2})}{P_{t+2}}$$

$$\Rightarrow \quad \frac{u'(C_{t+1})}{1+i_{t+1}} = \beta(1+r_{t+2}) \frac{u'(C_{t+2})}{1+i_{t+2}}$$

since $\frac{P_{t+2}}{P_{t+1}} = \frac{1+i_{t+2}}{1+r_{t+2}}$

- effective price of consumption is $1+i$, not 1.

From above, we have (using ($M_{t+1}$) and ($B_{t+1}$)):

$\mu_{t+1} = i_{t+1} \lambda_{t+1}$

- if $i>0$, the shadow value of the CIA constraint is positive $\Rightarrow$ CIA will bind

$\Rightarrow \frac{M_t + X_t}{P_t} = C_t$ ◦ demand for money.

- no interest elasticity in the demand for money

In equilibrium:

- $P_t F_N(K_t, N_t) = W_t$
- $F_K(K_t, N_t) = r_t + \delta$
- $\Pi_t = 0$
- $N_t = 1$
- $M_{t+1} - M_t = X_t$
  - gov't can increase money in this way.
- $B_{t+1} = B_t = 0$

Plugging these in:

$$P_t C_t + P_t K_{t+1} = \underbrace{P_t F_N(K_t, 1)}_{W_t} + \underbrace{(1 + F_K(K_t, 1) - \delta)}_{1+r_t} P_t K_t$$

$\Rightarrow K_{t+1} = F(K_t, 1) + (1-\delta)K_t - C_t$

Thus,

$$\frac{U'(C_{t+1})}{1+i_{t+1}} = \beta(1+r_{t+2})\frac{U'(C_{t+2})}{1+i_{t+2}}$$

$$(1+i_t) = (1+r_t)(1+\pi_t)$$

$$(1+r_t) = 1-\delta+F_K(K_t,1)$$

$$\frac{M_t+X_t}{P_t} = C_t$$

$$K_{t+1} = F(K_t,1)+(1-\delta)K_t - C_t$$

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Steady state:

Let $x$ - growth of money: $\frac{X_t}{P_t} = \frac{M_{t+1}-M_t}{P_t} = x\frac{M_t}{P_t}$

$$\Rightarrow x = \frac{M_{t+1}-M_t}{M_t} = \pi_{t+1}$$

From FOC: $(1+r) = 1+F_K(K,1)-\delta = 1/\beta$

from modified golden rule:

$\Rightarrow$ same level of capital

- $C = F(K,1) - \delta K$

In terms of real quantities, everything will look the same. (Superneutrality)

Thus, $i \approx \pi + r = x + r$. Fisher effect.

(from: $(1+i_t) = (1+r_t)(1+\pi_t)$)

Dynamics:

- Unexpected permanent increase in money growth leads to proportional increase in price level
  $\Rightarrow$ No real changes.

Nothing we can really do to have consumption respond to growth in money? Not true.

Temporary increase in money growth $\Rightarrow$ increase in interest rates $\Rightarrow$ decrease in consumption.

$\Rightarrow$ Wrong direction!

- Don't have neutrality for temporary changes in money growth.

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Money in the utility function

$$\max \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, \frac{M_{t+i}}{P_{t+i}})$$

$$\text{s.t. } P_t C_t + M_{t+1} + B_{t+1} = W_t + \Pi_t + M_t + (1+i_t)B_t + X_t$$

- $U_m > 0$, $U_{mc} \geq 0$.
  - having money makes consumption easier
    $\Rightarrow$ more desirable.

FOCs:

$(C_t)$: $U_c\left(C_t, \frac{M_t}{C_t}\right) = \lambda_t P_t$

$(B_{t+1})$: $\lambda_t = \lambda_{t+1}\beta(1+i_{t+1})$

$(M_{t+1})$: $\lambda_t = \beta\left[\lambda_{t+1} + \underbrace{\frac{1}{P_{t+1}} U_m\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)}_{\text{increases marginal utility in the future}}\right]$

Intertemporal:

$$U_c\left(C_t, \frac{M_t}{P_t}\right) = \beta(1+r_{t+1})\, U_c\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)$$

Intratemporal:

$$\frac{U_m\left(C_t, \frac{M_t}{P_t}\right)}{U_c\left(C_t, \frac{M_t}{P_t}\right)} = i_t$$

opportunity cost of holding money is a function of <u>nominal</u>, not real, interest rates.

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assume;

$$U(C, M/P) = \ln C + a \ln \frac{M}{P}$$

$$\Rightarrow \quad \left.\begin{array}{l} \frac{1}{C_t} = \beta(1+r_{t+1})\frac{1}{C_{t+1}} \\ \text{and } \frac{M_t}{P_t} = a\left(\frac{C_t}{I_t}\right) \end{array}\right\}$$ very primitive ISLM model.

Effects of money on steady state:

- $1 + F_K(K,1) - \delta = 1/\beta$ $\Rightarrow$ determines $r$
- $C = F(K,1) - \delta K$ $\Rightarrow$ determines $C$
- $\dfrac{U_m(C, \frac{M}{P})}{U_c(C, \frac{M}{P})} = i = \underbrace{x}_{\text{rate of money growth.}} + r$ $\Rightarrow$ determines $\frac{M}{P}$

- Money has no effect on real allocation.

What should the government do wrt optimal money growth?

- drive $U_m(C, \frac{M}{P})$ to zero

$\Rightarrow i \to 0 \Rightarrow x = -r$

- nominal interest rate zero gives no opportunity cost of money.
- in this model, deflation is good.
- No liquidity trap. Otherwise, this would be bad.