RBCs: (initially due to Prescott)
Shocks, Ramsey model, endogenous labor
Can replicate the correlations b/t output, consumption, and investment.

Problems:

1. Labor supply elasticity. To fit the data, these elasticities are not consistent with micro data.
2. Productivity shocks necessary to generate correlations would have to occur at high frequency in large magnitude.
3. Ignores money, which empirically seems to matter.

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Social planner:

$$\max E\left[\sum_{i=0}^{\infty} \beta^i U(C_{t+i}, L_{t+i}) \mid \Omega_t\right]$$

$$\text{s.t.} \quad N_{t+i} + L_{t+i} = 1$$

$$C_{t+i} + S_{t+i} = Z_{t+i} F(K_{t+i}, N_{t+i})$$

$$K_{t+i+1} = (1-\delta) K_{t+i} + S_{t+i}$$

We will ignore growth here.
Combining the constraints

$$K_{t+i+1} = (1-\delta)K_{t+i} + Z_{t+i} F(K_{t+i}, 1-L_{t+i}) - C_{t+i}$$

$$\mathcal{L} = E\big[U(C_t, L_t) - \lambda_t (K_{t+1} - (1-\delta)K_t - Z_t F(K_t, 1-L_t) + C_t) + \beta U(C_{t+1}, L_{t+1}) - \beta\lambda_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - Z_{t+1} F(K_{t+1}, 1-L_{t+1})) + \ldots \mid \Omega_t\big]$$

FOCs:

$(C_t)$: $U_c(C_t, L_t) = \lambda_t$

$(L_t)$: $U_L(C_t, L_t) = \lambda_t Z_t F_N(K_t, 1-L_t)$

$(K_{t+1})$: $\lambda_t = E[\beta \lambda_{t+1}(1-\delta + Z_{t+1} F_K(K_{t+1}, 1-L_{t+1})) \mid \Omega_t]$

Let $R_{t+1} = 1-\delta + Z_{t+1} F_K(K_{t+1}, 1-L_{t+1})$

$W_t = Z_t F_N(K_t, 1-L_t)$, so that we have:

$(C_t)$: $U_c(C_t, L_t) = \lambda_t$

$(L_t)$: $U_L(C_t, L_t) = \lambda_t W_t$

$(K_{t+1})$: $\lambda_t = E[\beta \lambda_{t+1} R_{t+1} \mid \Omega_t]$

$\Rightarrow$ intratemporal:
- $U_L(C_t, L_t) = W_t U_c(C_t, L_t)$

intertemporal:
- $U_c(C_t, L_t) = E[\beta R_{t+1} U_c(C_{t+1}, L_{t+1}) \mid \Omega_t]$

<u>What</u> restrictions on utility does balanced growth impose?

Hours worked has declined since the 1960s.
- BGP assumption seems to have flaws
  - Prescott says this is consistent with changes in tax rates.

BGP assumptions.

What if utility is:
- $U(C_1 + C_2, L_1 + L_2) + \beta^2 U(C_3 + C_4, L_3 + L_4) + \ldots$
- this has much different short-run implications than $\sum_{i=0}^{\infty} \beta^i U(C_i, L_i)$

For now, we impose BGP

Harrod neutral technology: $Y_t = Z_t F(K_t, A_t N_t)$, $A_t = A^t$, $A>1$

FOCs:

intratemporal:

$$\frac{U_L(CA^t, L)}{U_c(CA^t, L)} = wA^t$$

∘ $C, L, w$ constant over time

$\Rightarrow$ at $t=0$, $$\frac{U_L(C,L)}{U_c(C,L)} = w$$

$$\Rightarrow \frac{U_L(CA^t,L)}{U_c(CA^t,L)} = wA^t$$

∘ the MRS must be changing, or else L would change

Let $A^t = \frac{1}{C}$ (this expression needs to hold for any value of $A^t$)

$$\Rightarrow \frac{U_L(1,L)}{U_c(1,L)} = \frac{1}{C} \overbrace{\frac{U_L(C,L)}{U_c(C,L)}}^{=w}$$

$$\Rightarrow \frac{U_L(C,L)}{U_c(C,L)} = C\left[\frac{U_L(1,L)}{U_c(1,L)}\right]$$

For this, we need that the utility function is of the form $u(C\tilde{v}(L))$

∘ otherwise, people will take more leisure as technology increases.

intertemporal: (assume no uncertainty)

$$U_c(CA^t, L) = (\beta R)\, U_c(CA^{t+1}, L)$$

Imposing $U(C,L) = u(C\tilde{v}(L))$,

$$\frac{u'(CA^t\tilde{v}(L))}{u'(CA^{t+1}\tilde{v}(L))} = \beta R$$

For LHS to be constant, we need that the elasticity of intertemporal substitution is constant. That is,

$$u(C\tilde{v}(L)) = \begin{cases} \frac{\sigma}{\sigma-1}(C\tilde{v}(L))^{\frac{\sigma-1}{\sigma}} & \sigma \neq 1 \\ \log C + \underbrace{v(L)}_{\log \tilde{v}(L)} & \sigma = 1 \end{cases}$$

- $v$ needs to be concave

Special cases:

Prescott: $\log C + \varphi \log L$

New-Keynesian: $\log C - \frac{\psi}{1+\varphi} N^{1+\varphi}$

- constant elasticity of labor supply

FOCs with $U(C,L) = \log C + v(L)$

Intratemporal:

$$v'(L_t) = \frac{W_t}{C_t} = \underbrace{W_t}_{\text{substitution effect}} \cdot \underbrace{\frac{1}{C_t}}_{\text{income effect}}$$

$\frac{1}{C_t} = u' =$ marginal utility of wealth

Intertemporal:

$$E\left[\beta R_{t+1} \frac{C_t}{C_{t+1}} \Big| \Omega_t\right] = 1$$

Effects of a positive shock to $Z_t$

Consumption:
- smoothing ($C \uparrow$)
- tilting ($C \downarrow$)

Leisure / work:
- substitution: $W_t \uparrow \Rightarrow$ work harder
- income effect: $W_t \uparrow \Rightarrow$ consume more and enjoy more leisure

Depends on strength of these two effects

The more transitory the shock, the less strong the income effect, so substitution effect changes.

Employment effects

- use $v(L) = \varphi \log L \Rightarrow v'(L) = \varphi / L$

$\Rightarrow \varphi C_t = W_t L_t$ (intratemporal) intertemporal:

$$E\left[\beta\left(R_{t+1}\frac{W_t}{W_{t+1}}\right)\frac{L_t}{L_{t+1}} \mid \Omega_t\right]=1$$

Let $K_{t+1}=Z_t K_t^{\alpha}(1-L_t)^{1-\alpha}-C_t$

$u(C_t,L_t)=\log C_t+\varphi\log L_t$

$\delta=1$

$\Rightarrow$ N is constant by implication now. Income and substitution effects cancel

- $C_t=(1-\alpha\beta)Y_t$
- $I_t=\alpha\beta Y_t$

$\Rightarrow$ Cannot explain fluctuations in unemployment
Need a richer specification, which we can only solve numerically.