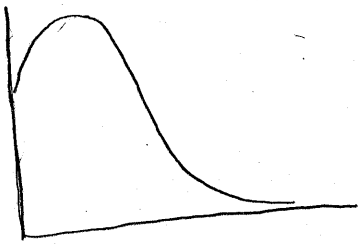
GDP:

y_t - log deviation from trend

Good representation is an AR(2) process

$$y_t = \underbrace{c_t}_{\text{will be small}} + 1.3 y_{t-1} - 0.4 y_{t-2} + \varepsilon_t$$



Trace effect of a shock

Alternatively, estimate:

$$\Delta y_t = c_t + \beta \Delta y_{t-1} + \varepsilon_t \text{, get}$$

$$y_t - y_{t-1} = c_t + 0.3(y_{t-1} - y_{t-2}) + \varepsilon_t$$

- no return to any trend
- unit root
- very different interpretation of the world

Co-movements of output with components

- Look at correlations between cyclical components of output and other variables
  - $\rho(\underbrace{X_{c,t}}_{\text{cyclical}}, Y_{c,t+k})$ $k = -6, -5, \ldots, 5, 6$

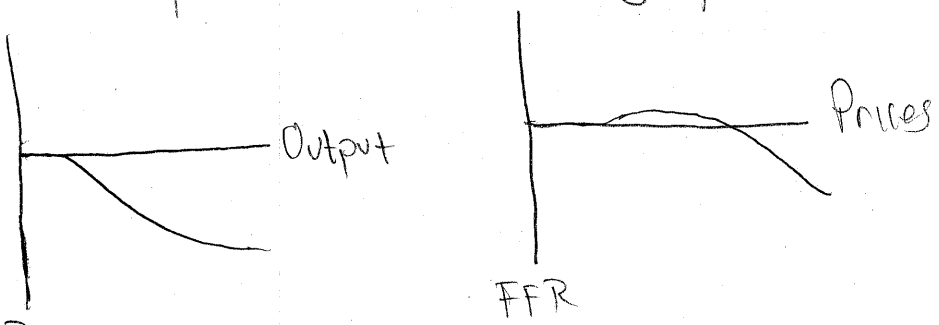
If $\rho(X_{ct}, Y_{c,t-6}) = 1$, then X is procyclical, and X lags Y

Facts:

1] output and consumption is highly correlated
2] output and investment is highly correlated.
3] output and inventory investment is highly correlated
   - would perhaps expect the opposite
4] output and exports is not very correlated
5] output and government spending is not very correlated
6] output and essentially all sectors are highly correlated.
7] output and employment is highly correlated
8] output and hours per worker is also highly correlated (over-time initially, then hire more workers)
9] output and productivity (Solow residual) quite correlated.
   - this seems problematic
10] output and real wage slightly positively correlated, but pretty much uncorrelated
11] output and real interest rate are negatively correlated. output and nominal interest rate is kinda correlated.
12] output and inflation are contemporaneously uncorrelated. Inflation definitely lags output.
   - This is similar to Phillips curve, because output and unemployment are very negatively correlated!

[3] Output and money are quite highly correlated.
How do we measure the causal relationship here?
Need exogenous shocks to money supply.

Impulse responses of monetary policy



FFR
Federal funds rate

Shocks and propagation mechanisms
Two ingredients:
[1] Shocks (productivity shocks.) Taste shocks don't work well
[2] Basic choice model b/t consumption/saving.
Ramsey model with technological shocks.
no heterogeneity, no movements in employment
no money, infinite horizon.

$$\max E\left[\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) \mid \Omega_t\right]$$

$$\text{s.t. } C_{t+i} + S_{t+i} = \underbrace{Z_{t+i}}_{\text{shock}} F(K_{t+i}, 1)$$

$$K_{t+i+1} = (1-\delta)K_{t+i} + S_{t+i}$$

Central planning problem

Combining constraints:

- $K_{t+i+1} = (1-\delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1) - C_{t+i}$

$$\mathcal{L} = E\Big[U(C_t) + \beta U(C_{t+1}) - \lambda_t(K_{t+1} - (1-\delta)K_t - Z_t F(K_t, 1)) + (t) - \beta\lambda_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - Z_{t+1}F(K_{t+1}, 1)) + (t+1) + \ldots \mid \Omega_t\Big]$$

FOCs:

(C_{t+i}): $E[U'(C_{t+i}) - \lambda_{t+i} \mid \Omega_t] = 0$

(K_{t+i+1}): $E[\lambda_{t+i} - \beta\lambda_{t+i+1}(1-\delta + Z_{t+i+1}F_K(K_{t+i+1}, 1)) \mid \Omega_t] = 0$

Define $R_{t+i+1} = 1 - \delta + Z_{t+i+1}F_K(K_{t+i+1}, 1)$.

At time t $(i=0)$, C_t, λ_t known. We get

- $U'(C_t) = \lambda_t$
- $\lambda_t = E[\beta R_{t+1}\lambda_{t+1} \mid \Omega_t]$

$$\Rightarrow U'(C_t) = E[\beta R_{t+1} U'(C_{t+1}) \mid \Omega_t]$$

- Smoothing and <u>tilting</u> (substitution effect)