Plan

1. Quality competition model
2. Exam advice
3. Questions (and answers).

Quality competition

- Higher quality goods replace older goods

$$Y_t = \frac{1}{1-\beta}\left[\int_0^1 \underbrace{q(v,t)}_{\text{quality of int. output}} \underbrace{k(v,t)^{1-\beta}}_{\text{intermediate output}} dv\right] L^\beta$$

- weighted sum of Cobb-Douglas prod. fcns.

$$\Pi = \frac{1}{1-\beta}\left[\int_0^1 q(v,t)k(v,t)^{1-\beta}dv\right]L^\beta - \int_0^1 \underbrace{\chi(v,t)}_{\text{price of int. good } v} k(v,t)dv - w_t L_t$$

FOC:

$$(k(v,t)):\ q(v,t)k(v,t)^{-\beta}L^\beta = \chi(v,t)$$

$$\Rightarrow\ k(v,t) = \underbrace{\left[\frac{q(v,t)}{\chi(v,t)}\right]^{1/\beta}}_{\text{what matters is this quality/price ratio}} L$$

Suppose for some v, $q(v,t) \longrightarrow \lambda q(v,t)$, $\lambda > 1$

costs $\psi q(v,t)$ to produce a good at quality $q(v,t)$

- Best quality/price ratio can offer is $\frac{q}{\psi q}$
  - ψq is breakeven price

Old	New
q	λq
$\frac{q}{\psi q}$	$\frac{\lambda q}{\chi}$

- Can choose χ to displace the incumbent:
- What choice of χ should producer of v choose?

- will choose $\chi = \psi \lambda q$
- assume $\psi = \lambda^{-1} \Rightarrow \chi = q$
- $k(v,t) = \left[\frac{q(v,t)}{\chi(v,t)}\right]^{1/\beta} L = \left[\frac{q(v,t)}{q(v,t)}\right]^{1/\beta} L = L$

$$\Rightarrow Y(t) = \frac{1}{1-\beta}\left[\int_0^1 q(v,t)\, L^{1-\beta}\, dv\right] L^{\beta}$$

$$= \frac{1}{1-\beta}\left[\int_0^1 q(v,t)\, dv\right] L$$

$$= \frac{1}{1-\beta} Q(t) L \quad \text{where } \int_0^1 q(v,t)\,dv = Q(t)$$

Then $rV - \dot{V} = \pi - \underbrace{x}_{\text{rate at which new innovations arrive}} V$ - innovation sector

$$\frac{\dot{Q}}{Q} = \frac{\int_0^1 \dot{q}(v)\, dv}{\int_0^1 q(v)\, dv} \equiv \frac{\int_0^1 (\lambda - 1)\, x\, q(v)\, dv}{\int_0^1 q(v)\, dv} = (\lambda - 1)\, x$$

- In steady state, $\dot{V}=0$, so that

$$rV = \pi - xV \Rightarrow \underbrace{V}_{\text{value of innovation in the R\&D sector}} = \frac{\pi^*}{r^*+x^*} = \frac{\pi^*}{r^* + \frac{g^*}{\lambda-1}}$$, where

g^* is the growth rate of the economy on the balanced growth path.

$$g^* = \frac{\dot{c}}{c} = \frac{1}{\theta}(r^* - \rho)$$

1. Final goods → Demand for v
2. R & D sector → V, r^*
3. Intermediate good sector → Prices, quantities

Free entry in R&D: $qz = \mu z V$

In these models, it is not clear whether there is too much or too little innovation.

- In expanding variety model, there is always too little innovation
- Here, there are two opposing forces:
  - standing on shoulders, and
  - stepping on toes

Exam:

Essential:
- 9.2 – 9.4 (also, maybe 9.5)
- 14.1 (quality competition)
- 15.1 – 15.2 of old notes

Recommended: 5 (NC growth), 7 (optimal control), 8 (Ramsey), 11, 13, 14

Exam:

- 1 long question
  - optimization techniques (be quick at this)
    - read Suman's handout!
  - Discrete and continuous time.

$$\frac{c_{t+1}}{c_t} = [\beta(1+A-\delta)]^{\theta}$$

- Euler equation in discrete time

$$\ln c_{t+1} - \ln c_t = \theta \ln \beta + \theta \ln(1+A-\delta)$$

- $(e^{-\rho})^t = \beta^t \iff \beta = e^{-\rho}$

$$\Rightarrow \ln c_{t+1} - \ln c_t \approx -\theta\rho + \theta \ln(1+A-\delta)$$

$$\approx -\theta\rho + \theta(A-\delta)$$

$$\frac{\dot{c}}{c} \approx \theta(A-\delta-\rho)$$

- $\rho = \frac{1}{\beta} - 1 \Rightarrow \beta = \frac{1}{1+\rho}$

$$\Rightarrow \log \beta = -\log(1+\rho) \approx -\rho$$

$$\Rightarrow \beta \approx e^{-\rho}$$

- Phrase diagrams
- 4 short questions
  - Basic analytical techniques
  - Normative and positive implications
    - ie optimal tax
  - Empirical

ie: In model w/expanding variety, if you put a subsidy in the R&D sector, can you achieve first best?

Characterize:
- transversality
- Euler equation

} necessary and sufficient conditions under concavity