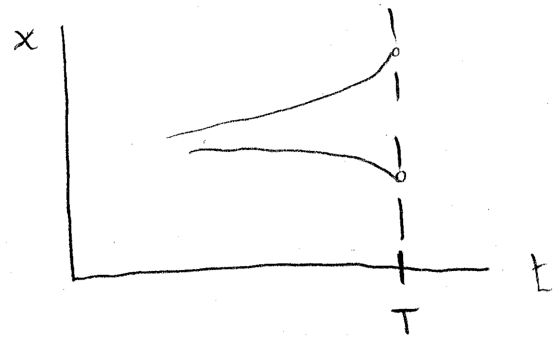
Plan:

1. Optimization handout
2. Overlapping generations
3. Exam guidance?

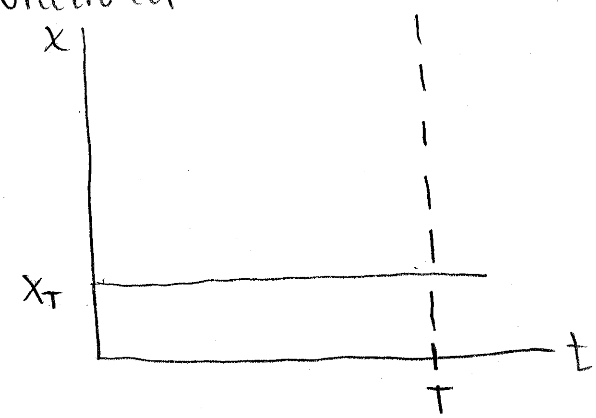
- Vertical terminal line



transversality condition:
$\lambda(T) = 0$

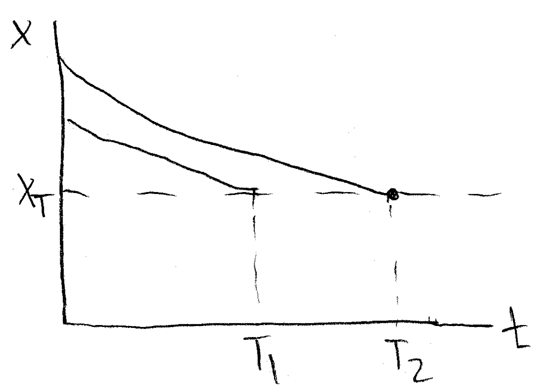
- ie $x(T)$ should be such that you consume everything by the last period

- Truncated



- if there is a lower bound. Complementary slackness condition:
$\lambda(T) \geq 0$, $x(T) \geq x_T$
and $\lambda(T)(x(T) - x_T) = 0$

- Horizontal terminal line



- x_T fixed - can stop whenever you want

$H(T, x(T), y(T), \lambda(T)) = 0$

- If want to maximize integral, need to pick pt where $H = 0$.

Exam guidance:

- make sure we are comfortable with the optimization handout
- be comfortable with Ramsey setup
- phase diagrams, shifts in phase diagrams
- understand the intuition behind consumption smoothing (ie applications to tax smoothing)
  - equate MU's.
- Few (2-3) short questions
- Two long questions
- Understand basic models and mechanisms in the endogenous growth model.

Overlapping generations and growth

- Finitely lived consumers

(*) Economy

- individuals live for 2 periods:

$$\underbrace{U(t)}_{\text{born at time } t} = u(c_1(t)) + \beta u(c_2(t+1))$$

Work and consume when young; supply 1 unit of labor inelasticly (slavery?) Get wage $w(t)$

Old: consume only

$L(t) = L(0)(1+n)^t$

Competitive equilibrium (Production side is same)

Households born at t:
$$\max_{c_1(t),c_2(t+1),s(t)} u(c_1(t)) + \beta u(c_2(t+1))$$

s.t.
$$c_1(t) + s(t) \le w(t) \qquad (\lambda_1)$$
$$c_2(t+1) \le R(t+1)s(t) \qquad (\lambda_2)$$

FOC:

$(c_1(t))$: $u'(c_1(t)) = \lambda_1$

$(c_2(t+1))$: $\beta u'(c_2(t+1)) = \lambda_2$

$(s(t))$: $-\lambda_1 + R(t+1)\lambda_2 = 0$

$$\Rightarrow \frac{u'(c_1(t))}{\beta u'(c_2(t+1))} = R(t+1)$$

Firms: (full depreciation)

$R(t) = f'(k(t))$

$w(t) = f(k(t)) - k(t)f'(k(t))$

Competitive equilibrium: $\{K(t), c_1(t), c_2(t+1), R(t), w(t)\}_{t=0}^{\infty}$ such that

- Households maximize given prices
- Firms do
- Markets clear

Dynamics

$$k(t+1) = \frac{K(t+1)}{L(t+1)} = \frac{s(t)L(t)}{L(t+1)} = \frac{s(t)}{1+n}$$

$$= \frac{s(w(t), R(t+1))}{1+n}$$

since, due to full depreciation, $K(t+1) = s(t)L(t)$

$$\Rightarrow k(t+1) = \frac{s(f(k(t)) - k(t)f'(k(t)), f'(k(t+1)))}{1+n}$$

Steady state:

$$k^* = \frac{s(f(k^*) - k^* f'(k^*), f'(k^*))}{1+n}$$

Could look strange:

With log preferences

$U(t) = \log c_1(t) + \beta \log c_2(t+1)$

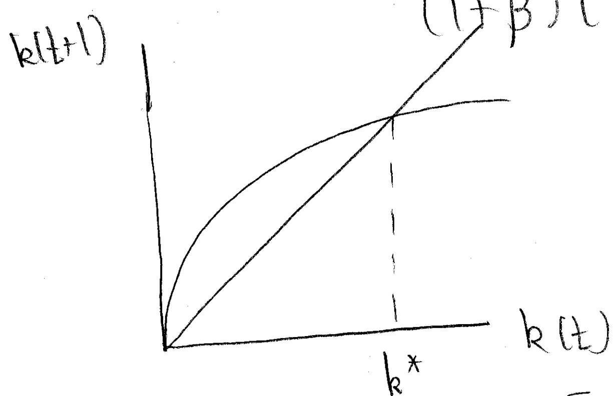
$f(k(t)) = k(t)^\alpha$

Then:
$$\frac{c_2(t+1)}{c_1(t)} = \beta R(t+1)$$
$$\Rightarrow \frac{c_2(t+1)}{R(t+1)} = \beta c_1(t)$$

Since $c_2(t+1) = R(t+1)s(t)$, we have:

$$c_1(t) + \frac{c_2(t+1)}{R(t+1)} = w(t)$$
$$\Rightarrow c_1(t)(1+\beta) = w(t)$$
$$\Rightarrow c_1(t) = \frac{w(t)}{1-\beta}, \quad s(t) = \frac{\beta}{1+\beta} w(t)$$

Then
$$k(t+1) = \frac{\beta w(t)}{(1+\beta)(1+n)} = \frac{\beta (k^\alpha - k\alpha k^{\alpha-1})}{(1+\beta)(1+n)}$$
$$= \frac{\beta (1-\alpha) k(t)^\alpha}{(1+\beta)(1+n)}$$



Steady state: $k^* = \left[\frac{\beta(1-\alpha)}{(1+\beta)(1+n)}\right]^{\frac{1}{1-\alpha}}$

Pareto Optimality $\quad \beta_s \neq \beta$ necessarily

$$\max \sum_{t=0}^{\infty} \beta_s^t \left[u(c_1(t)) + \beta u(c_2(t+1)) \right]$$

$$\text{s.t. } F(K(t), L(t)) = K(t+1) + L(t)c_1(t) + L(t-1)c_2(t)$$

$$\Rightarrow f(k(t)) = (1+n)k(t+1) + c_1(t) + c_2(t) \cdot \frac{1}{1+n} \qquad t = 0, 1, \ldots$$

FOC:

$$(c_1(t)): \ \beta_s^t u'(c_1(t)) = \beta_s^t \lambda_t$$

$$(c_2(t+1)): \ \beta_s^t \beta u'(c_2(t+1)) = \beta_s^{t+1} \frac{\lambda_{t+1}}{1+n}$$

$$(k(t+1)): \ -\beta_s^t (1+n)\lambda_t + \beta_s^{t+1} \lambda_{t+1} f'(k(t+1)) = 0$$

$$\Rightarrow \frac{u'(c_1(t))}{\beta u'(c_2(t+1))} = \frac{\beta_s \lambda_t (1+n)}{\lambda_{t+1}} = f'(k(t+1))$$