Ricardian equivalence:

- equivalence between lump-sum taxes and government bonds in an infinite horizon problem with complete markets
- changing timing of taxes without changing present value of taxes does not change consumer choices.

Without complete markets (ie no borrowing), the two are clearly not equivalent:

- if tax today consumers would like to borrow to smooth consumption

Endogenous Growth

- $Y_t = F(K_t, L_t) = AK_t$, $A > 0$

$\Rightarrow y_t = Ak_t$ · MPK is not decreasing

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t$$

$$\Rightarrow \frac{c_{t+1}}{c_t} = [\beta(1+A-\delta)]^\theta \quad \text{with CEIS}$$

$$\Rightarrow \ln c_{t+1} - \ln c_t = \theta(A - \delta - \rho)$$

Resources grow linearly with k_t

$$c_t + k_{t+1} = (1+A-\delta)k_t$$

Guess that $c_t = (1-s)(1+A-\delta)k_t$

$$k_{t+1} = s(1+A-\delta)k_t$$

- want to find s.

Verify that this satisfies the Euler condition

We have that $\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t}$

$$\frac{c_{t+1}}{c_t} = [\beta(1+A-\delta)]^\theta > 0 \quad \text{iff} \quad \beta(1+A-\delta) > 1$$

$$\text{iff} \quad A - \delta > \frac{1}{\beta} - 1$$

- impose this cond.

Resource constr.

$$\frac{c_t}{k_t} + \underbrace{\frac{k_{t+1}}{k_t}}_{= \frac{c_{t+1}}{c_t} = [\beta(1+A-\delta)]^\theta} = (1+A-\delta)$$

$$\Rightarrow \underset{\substack{\parallel \\ \frac{1-s}{s}}}{\frac{c_t}{k_t}} = (1+A-\delta) - [\beta(1+A-\delta)]^\theta$$

$$\Rightarrow s = \beta^\theta (1+A-\delta)^{\theta-1}$$

s is $\uparrow$ in β

s is $\uparrow$ in A if $\theta > 1$

$\downarrow$ in A if $\theta < 1$

$\frac{c_{t+1}}{c_t}$ $\uparrow$ in A unambiguously

- suppose $\theta > 1$. If A very large, $s > 1$
  - $\max \sum \beta^t c_t$
    - s.t. $c_t + k_{t+1} = R k_t$
    - if $R < 1$, consume all at $t=0$
      - $R = 1$, consume whenever
      - $R > 1$, save forever
    - extreme case where $\theta = +\infty > 1$.
  - ensuring that A is sufficiently small is equivalent to ensuring that our transversality is satisfied.

Obviously, the competitive equilibrium will coincide with the solution to the social planner's problem, since there are no externalities.

Now, we will think of different types of capital and we will endogenize A.

The essence of the ability to have sustained growth is the linearity of output with respect to the accumulable factors.

$Y_t = F(K_t, H_t) = F(K_t, h_t L_t)$ CRS

$y_t = \frac{Y_t}{L_t} = F(k_t, h_t)$, which is still CRS.

RC: $c_t + i_t^k + i_t^h \le y_t$

$k_{t+1} = (1-\delta_k) k_t + i_t^k$

$h_{t+1} = (1-\delta_h) h_t + i_t^h$

$\Rightarrow c_t + k_{t+1} + h_{t+1} \le F(k_t, h_t) + (1-\delta_k) k_t + (1-\delta_h) h_t$

Two Euler equations:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta [1 + F_k(k_{t+1}, h_{t+1}) - \delta_k]$$

and $$\frac{u'(c_t)}{u'(c_{t+1})} = \beta [1 + F_h(k_{t+1}, h_{t+1}) - \delta_h]$$

By CRS, $y_t = F(k_t, h_t) = h_t F\left(\frac{k_t}{h_t}, 1\right) = \frac{h_t}{k_t + h_t} (k_t + h_t) F\left(\frac{k_t}{h_t}, 1\right)$

$y_t = F(k_t, h_t) = A(\kappa_t)(k_t + h_t)$ $\qquad \kappa_t = \frac{k_t}{h_t}$

$A(\kappa) = \frac{F(\kappa, 1)}{1+\kappa} = \frac{f(\kappa)}{1+\kappa}$

◦ Now, this is in an AK form as above

Irreversible investment leads to accelerated growth.