Plan:

1] PS2: Qs 1, 3

2] aggregation for prod fcns

3] MATLAB: Method + Justification

CE: {prices, quantities} s.t.

1] Households maximize U_0 s.t. intertemporal budget constraint and no Ponzi conditions, taking prices as given

2] Firms maximize Πs s.t. technology and prices

3] Markets clear: $a(t)=K(t)$ + capital accumulation equation.

MATLAB

$$v(k) = \max_{\underbrace{k' \in [0, f(k)+(1-\delta)k]}_{\equiv \Gamma(k)}} \{u(c) + \beta v(k')\}$$

$$\text{s.t. } c + k' \leq f(k) + (1-\delta)k$$

$$c, k' \geq 0$$

$$\Leftrightarrow v(k) = \max_{k' \in \Gamma(k)} \{u(f(k)+(1-\delta)k-k') + \beta v(k')\}$$

Define the operator

$$v_{j+1}(k) = T(v_j(k)) = \max_{k' \in \Gamma(k)} \{u(f(k)+(1-\delta)k-k') + \beta v_j(k')\}$$

- If $v^*(k) = T(v^*(k))$, then we have a fixed point, and this $v^*(k)$ is our optimal value function
- By the contraction mapping theorem, if T is a contraction, then $v_{j+1}(k) \to v^*(k)$ $\forall k$.
- Given a function v, we can derive the optimal policy function.

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 2 \\ 6 & 0 & 0 \end{bmatrix}$$

$\max(X) = [6 \quad 2 \quad 3]$ ◦ maximizes over each column

$a = [0 : 0.1 : 1]$

$> a = [0 \quad 0.1 \quad 0.2 \quad \cdots \quad 0.9 \quad 1]$

$$C = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} [f(k_1) \quad f(k_2) \cdots f(k_{1000})] - \begin{bmatrix} k_1 \\ \vdots \\ k_{1000} \end{bmatrix} [1 \cdots 1]$$

$$= \overbrace{\begin{bmatrix} f(k_1) - k_1 & \cdots & f(k_{1000}) - k_1 \\ \vdots & & \\ f(k_1) - k_{1000} & \cdots & f(k_{1000}) - k_{1000} \end{bmatrix}}^{\text{today's } k} \Big\} \text{tomorrow's capital}$$

(*) Here, f is net of depreciated capital

$\max(C)$ gives best choice of tomorrow's capital as a fcn of today's capital.

$\sim C = [1 \text{ if } c = 0 \quad 0 \text{ if } c \neq 0]$