

Let $f(k)$ be s.t.

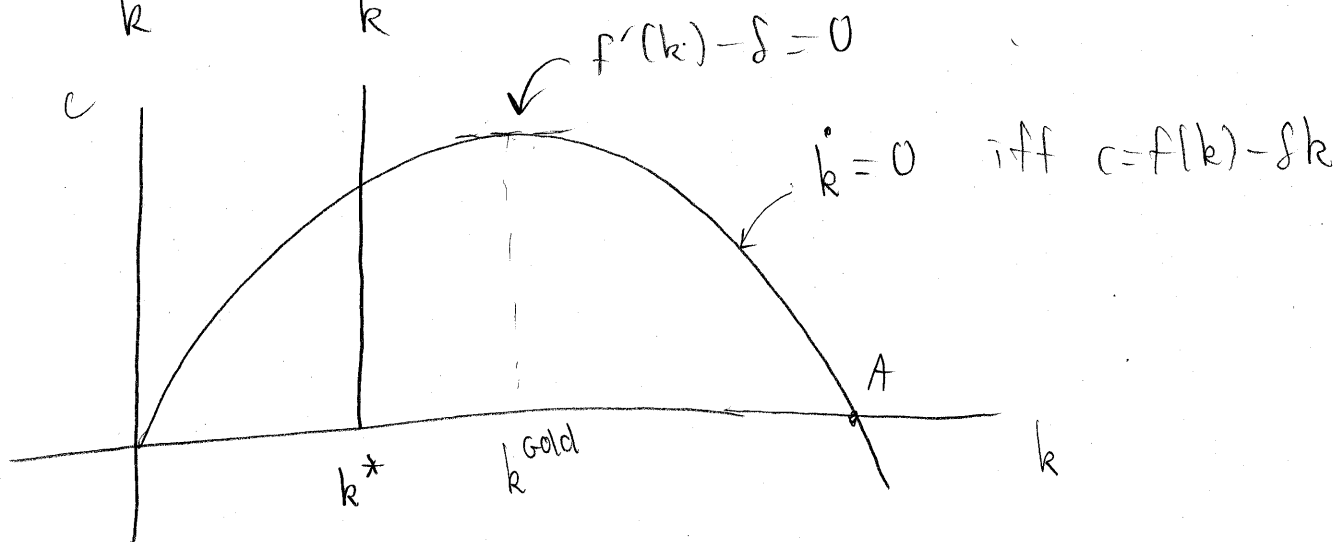
$$f(k) = k^\alpha + A$$

$$f'(k) = \alpha k^{\alpha-1} \neq g(A)$$

$$\frac{f(k)}{k} = \frac{k^\alpha + A}{k} \uparrow \text{ in } A \Rightarrow s \downarrow \text{ in } A.$$

$$c = f(k) - sk$$

$$s = \frac{sk}{y} = s \cdot \frac{1}{f(k)/k} = g\left(\frac{f(k)}{k}\right)$$



• A is not optimal, because we can go to c^{Gold} and consume more in every single future period.

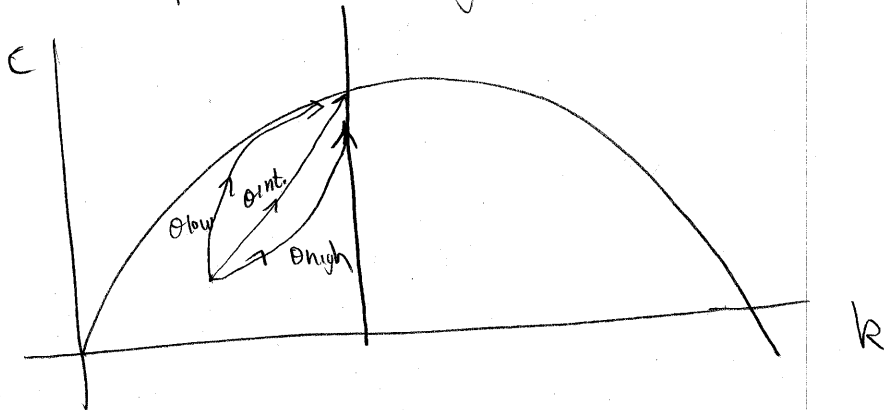
$$U_0 = u_0 + \beta u_1 + \beta^2 u_2 + \beta^3 u_3 + \dots$$

$$U_1 = u_1 + \beta u_2 + \beta^2 u_3 + \dots$$

• What if SP maximizes U_1 instead of U_0 ?
 • still have Pareto Optimality. (Will be a corner solution)

In the phase diagrams, phase boundaries do not change as θ changes

Optimal path changes with θ :



Recall:

$$\dot{c} = \theta c [f'(k) - \delta - \rho]$$

$$\dot{k} = f(k) - \delta k - c$$

$$\Rightarrow \frac{\dot{c}}{\dot{k}} = \frac{dc}{dk} = \frac{\theta c [f'(k) - \delta - \rho]}{f(k) - \delta k - c}$$

◦ differential equation in c as a function of k .

◦ family of solutions.

◦ optimal solution will cross the steady state.

◦ at steady state, $\frac{dc}{dk} = \frac{0}{0}$. Apply L'Hopital, get two solutions. One is stable, the other is not.

endowment $e > 0$ exogenous. What happens?

$$c_t^j + k_{t+1}^j = w_t + r_t k_t^j + (1-\delta)k_t^j + e$$

$$k_{t+1} - k_t = f(k_t) - \delta k_t - c_t + e$$

- c equation doesn't change
- k shifts upward

Alternatively, consider a linear income tax transferred as lump sums back to consumers.

