Growth

- Daron's textbook is available on his website.

Facts:

- From 1870-2000, US per-capita GDP grew at about 1.75% per year. Grew by about 10x during this period.
  - what can we do about increasing growth rates?

Solow model:

- Behavioral assumption: people save a constant, exogenous fraction of their wealth each period.
  - the Neoclassical growth model puts microfoundations into this type of model.

- $Y_t = F(K_t, L_t)$
  - $F: \mathbb{R}_+^2 \to \mathbb{R}_+$ is a (stationary) production function
  - assume $F \in C^2$
    - assume L_t grows at a constant rate.
- F is neoclassical if $F(\mu K, \mu L) = \mu F(K, L)$
- $\underbrace{\frac{\partial F}{\partial L}}_{F_L} > 0$, $\underbrace{\frac{\partial F}{\partial K}}_{F_K} > 0$, $\underbrace{\frac{\partial^2 F}{\partial L^2}}_{F_{LL}} < 0$, $\underbrace{\frac{\partial^2 F}{\partial K^2}}_{F_{KK}} < 0$
- $\lim_{K \to 0} F_K = \lim_{L \to 0} F_L = +\infty$

  $\lim_{K \to +\infty} F_K = \lim_{L \to \infty} F_L = 0$

Euler's thm: $Y = F(K, L) = K F_K + L F_L$ (divide by F)

$\iff 1 = \varepsilon_K + \varepsilon_L$

- In equilibrium, $F_K = r$, $F_L = w$

$$\Rightarrow 1 = \varepsilon_K + \varepsilon_L = \underbrace{\frac{rK}{F}}_{\text{capital's share}} + \underbrace{\frac{wL}{F}}_{\text{labor's share}}$$

assume capital and labor are essential: $F(0,L) = F(K,0) = 0$.

Divide everything by L: $y = \frac{Y}{L}$, $k = \frac{K}{L}$

$y = f(k) = F(k,1)$ (by homogeneity)

$\Rightarrow f'(k) > 0 > f''(k)$

$\lim_{k \to 0} f'(k) = +\infty$, $\lim_{k \to +\infty} f'(k) = 0$

$F_K = f'(k)$, $F_L = f(k) - f'(k)k$

Resource constraint and law of motions:

- $C_t + I_t \leq Y_t$

$\Rightarrow c_t + i_t \leq y_t$

- assume $\frac{L_{t+1}}{L_t} = 1+n$ $\forall t \Rightarrow L_t = (1+n)^t L_0$

Law of motion:

- $K_{t+1} = (1-\delta)K_t + I_t$

$\Rightarrow (1+n)k_{t+1} = (1-\delta)k_t + i_t$

$\Rightarrow k_{t+1} \approx (1 \underbrace{-\delta - n}_{\delta + n = \text{"effective" depreciation rate of capital}})k_t + i_t$

Thus, $k_{t+1} - k_t \leq f(k_t) - (\delta + n)k_t + c_t$

a feasible allocation is a sequence $\{c_t, k_t\}_{t=0}^{\infty}$ s.t.

$$k_{t+1} \leq f(k_t) + (1-\delta-n)k_t - c_t$$

assume $c_t = (1-s_t)f(k_t)$, $s \in (0,1)$

a plan is "optimal" if $c_t = (1-s_t)f(k_t)$, resource constraint holds with equality, and the allocation is feasible.

Define the policy rule: $k_{t+1} = G(k_t)$ to recursively give capital levels

- Here, $G(k) = sf(k) + (1-\delta-n)k$

The growth rate is thus $\gamma_t \equiv \frac{k_{t+1}-k_t}{k_t} \equiv \gamma(k_t)$

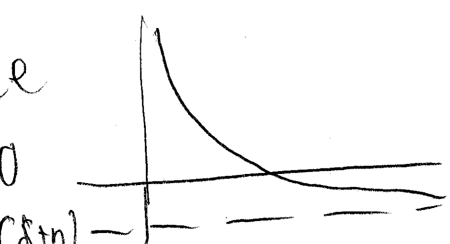
$$\Rightarrow \gamma(k) = s\frac{f(k)}{k} - (\delta+n) \equiv s\underbrace{\varphi(k)}_{\text{avg. output}} - \underbrace{(\delta+n)}_{\text{effective depreciation}}$$

Steady state: $k^{ss} = G(k^{ss})$ iff $\gamma(k^{ss}) = \frac{k^{ss}-k^{ss}}{k^{ss}} = 0$

$$\Leftrightarrow \quad 0 = s\frac{f(k^{ss})}{k^{ss}} - (\delta+n)$$

By intermediate value thm, since

$\lim_{k\to 0} \varphi(k) = +\infty$ and $\lim_{k\to+\infty} \varphi(k) = 0$



and $\varphi(k)$ is continuous, there is a soln. Since $\varphi'(k) = \frac{f'(k)k - f(k)}{k^2} = -\frac{F_L}{k^2} < 0$, it is unique.