14.451: Macroeconomic Theory I

Overview of Key Results: Chapter 2, Acemoglu

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1 Propositions, Definitions, etc.

Assumption 1'. The production function $F : \mathbb{R}_+^3 \to \mathbb{R}_+$ in (3.13) is twice continuously differentiable in K, H and L, and satisfies

$$\begin{aligned} \frac{\partial F(K,H,AL)}{\partial K} &> 0, \ \frac{\partial F(K,H,AL)}{\partial H} > 0, \ \frac{\partial F(K,H,AL)}{\partial L} > 0 \\ \frac{\partial^2 F(K,H,AL)}{\partial K^2} &< 0, \ \frac{\partial^2 F(K,H,AL)}{\partial H^2} < 0, \ \frac{\partial^2 F(K,H,AL)}{\partial L^2} < 0. \end{aligned}$$

Moreover, F exhibits constant returns to scale in its three arguments.

Assumption 2'. F satisfies the Inada conditions

$$\begin{aligned} \lim_{K\to 0} \frac{\partial F(K,H,AL)}{\partial K} &= \infty \text{ and } \lim_{K\to \infty} \frac{\partial F(K,H,AL)}{\partial K} = 0 \text{ for all } H > 0 \text{ and all } AL > 0, \\ \lim_{H\to 0} \frac{\partial F(K,H,AL)}{\partial H} &= \infty \text{ and } \lim_{H\to \infty} \frac{\partial F(K,H,AL)}{\partial H} = 0 \text{ for all } K > 0 \text{ and all } AL > 0, \\ \lim_{L\to 0} \frac{\partial F(K,H,AL)}{\partial L} &= \infty \text{ and } \lim_{L\to \infty} \frac{\partial F(K,H,AL)}{\partial L} = 0 \text{ for all } K, H, A > 0. \end{aligned}$$

Proposition 3.1. Suppose Assumptions 1' and 2' are satisfied. Then in the augmented Solow model with human capital, there exists a unique steady-state equilibrium (k^*, h^*).

Proposition 3.2. Suppose Assumptions 1' and 2' are satisfied. Then the unique steady-state equilibrium of the augmented Solow model with human capital, (k^*, h^*), is globally stable i nthe sense that starting with any $k(0) > 0$ and $h(0)$, we have $(k(t), h(t)) \to (k^*, h^*)$.

2 Equations

$$\frac{\dot{Y}}{Y} = \frac{F_A A}{Y}\frac{\dot{A}}{A} + \frac{F_K K}{Y}\frac{\dot{K}}{K} + \frac{F_L L}{Y}\frac{\dot{L}}{L} \tag{3.1}$$

$$x = g - \alpha_K g_K - \alpha_L g_L \tag{3.2}$$

$$\hat{x}(t) = g(t) - \alpha_K(t) g_K(t) - \alpha_L(t) g_L(t) \tag{3.3}$$

$$\hat{x}_{t,t+1} = g_{t,t+1} - \bar{\alpha}_{K,t,t+1} g_{K,t,t+1} - \bar{\alpha}_{L,t,t+1} \tag{3.4}$$

$$y(t) = A(t) f(k(t)) \tag{3.5}$$

$$\frac{\dot{k}(t)}{k(t)} = \frac{sf(k(t))}{k(t)} - \delta - g - n \tag{3.6}$$

$$\begin{aligned} \frac{\dot{y}(t)}{y(t)} &= g + \varepsilon_f(k(t))\frac{\dot{k}(t)}{k(t)}, \\ \varepsilon_f(k(t)) &= \frac{f'(k(t))\,k(t)}{f(k(t))} \in (0,1) \end{aligned} \tag{3.7}$$

$$\frac{\dot{y}(t)}{y(t)} \simeq g - (1 - \varepsilon_f(k^*))(\delta + g + n)(\log y(t) - \log y^*(t)) \tag{3.8}$$

$$g_{i,t,t-1} = b^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t} \tag{3.9}$$

$$g_{i,t,t-1} = b_i^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t} \tag{3.10}$$

$$g_{i,t,t-1} = \mathbf{X}'_{i,t}\beta + b^1 \log y_{i,t-1} + \varepsilon_{i,t} \tag{3.11}$$

$$\log y_{i,t} = \alpha \log y_{i,t-1} + \mathbf{X}'_{i,t}\beta + \delta_i + \mu_t + \varepsilon_{i,t} \tag{3.12}$$

$$Y = F(K, H, AL) \tag{3.13}$$

$$s_k f(k^*, h^*) - (\delta_k + g + n) k^* = 0 \tag{3.14}$$

$$s_h f(k^*, h^*) - (\delta_h + g + n) h^* = 0 \tag{3.15}$$

$$\left.\frac{dh}{dk}\right|_{\dot{k}=0} = \frac{(\delta_k + g + n) - s_k f_k(k^*, h^*)}{s_k f_h(k^*, h^*)} \tag{3.16}$$

$$\left.\frac{dh}{dk}\right|_{\dot{h}=0} = \frac{s_h f_k(k^*, h^*)}{(\delta_h + g + n) - s_h f_h(k^*, h^*)} \tag{3.17}$$

$$Y(t) = K^\beta(t) H^\alpha(t) (A(t) L(t))^{1-\alpha-\beta} \tag{3.18}$$

$$
\begin{aligned}
k^* &= \left(\left(\frac{s_k}{n+g+\delta_k}\right)^{1-\alpha}\left(\frac{s_h}{n+g+\delta_h}\right)^{\alpha}\right)^{\frac{1}{1-\alpha-\beta}} \\
h^* &= \left(\left(\frac{s_k}{n+g+\delta_k}\right)^{\beta}\left(\frac{s_h}{n+g+\delta_h}\right)^{1-\beta}\right)^{\frac{1}{1-\alpha-\beta}}
\end{aligned} \tag{3.19}
$$

$$
\hat{y}^* = \left(\frac{s_k}{n+g+\delta_k}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(\frac{s_h}{n+g+\delta_h}\right)^{\frac{\alpha}{1-\alpha-\beta}} \tag{3.20}
$$

$$
y_j^*(t) \equiv \frac{Y(t)}{L(t)} = A_j(t)\left(\frac{s_{k,j}}{n_j+g_j+\delta_k}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(\frac{s_{h,j}}{n_j+g_j+\delta_h}\right)^{\frac{\alpha}{1-\alpha-\beta}} \tag{3.21}
$$

$$
\ln y_j^*(t) = \ln \bar{A}_j + gt + \frac{\beta}{1-\alpha-\beta}\ln\left(\frac{s_{k,j}}{n_j+g+\delta_k}\right) + \frac{\alpha}{1-\alpha-\beta}\ln\left(\frac{s_{h,j}}{n_j+g+\delta_h}\right) \tag{3.22}
$$

$$
\begin{aligned}
\ln y_j^* &= \text{constant} + \frac{\beta}{1-\alpha-\beta}\ln(s_{k,j}) - \frac{\beta}{1-\alpha-\beta}\ln(n_j+g+\delta_k) \\
&\quad + \frac{\alpha}{1-\alpha-\beta}\ln(s_{h,j}) - \frac{\alpha}{1-\alpha-\beta}\ln(n_j+g+\delta_h) + \varepsilon_j
\end{aligned} \tag{3.23}
$$

$$
\ln w_i = \mathbf{X}_i'\gamma + \phi S_i \tag{3.24}
$$

$$
R_j = (1-\alpha)\left(\frac{K_f}{A_j H_f}\right)^{-\alpha} \tag{3.25}
$$

$$
Y_j = K_j^{1-\alpha}(A_j H_j)^{\alpha} \tag{3.26}
$$

$$
\hat{x}_{j,j+1} = g_{j,j+1} - \bar{\alpha}_{K,j,j+1} g_{K,j,j+1} - \bar{\alpha}_{L,j,j+1} g_{H,j,j+1} \tag{3.27}
$$

$$
\log Y_j = (1-\alpha)\log K_j + \alpha \log H_j + \alpha \log A_j \tag{3.28}
$$

$$
\begin{aligned}
X_j^K &= A_j^k H_j - c_j^s \sum_{i=1}^{N} A_i^k K_i \\
X_j^H &= A_j^h H_j - c_j^s \sum_{i=1}^{N} A_i^h H_i
\end{aligned} \tag{3.29}
$$

$$
\frac{R_j}{A_j^k} = \frac{R_{j'}}{A_{j'}^k} \tag{3.30}
$$

$$
\frac{w_j}{A_j^h} = \frac{w_{j'}}{A_{j'}^h} \tag{3.31}
$$