Haile and Tamer:

Recall:

$$\varphi\left(F_{b^{(n:n)}+\Delta}(v); n-1, n\right) \le F(v) \le \min_j \varphi\left(F_{b^{(j:n)}}(v); j, n\right)$$

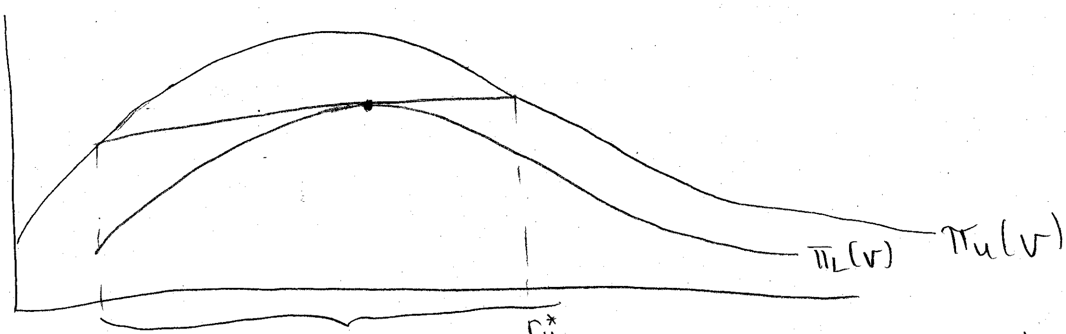
$$\Rightarrow \quad F_L(v) \le F(v) \le F_u(v)$$

Optimal reserve price maximizes

$$\pi(r) = (r-\underline{v})[1-F_v(v)] \qquad r \ge \underline{v}$$

$$\Rightarrow \pi_L(v) \le \pi(r) \le \pi_u(v), \text{ where}$$

$$\pi_L(v) = (r-\underline{v})(1-F_u(v)), \ \pi_H(v) = (r-\underline{v})(1-F_L(v))$$



the maximum must occur somewhere between r_l^* and r_u^*

- How do we do this in the data?

Estimation

- $T_n = \sum_{t=1}^{T} 1_{\{n_t = n\}}$ = # auctions with n bidders

- $\hat{G}_{i,n}(v) = \frac{1}{T}\sum_{t=1}^{T} 1_{\{n_t = n, b^{i,n_t} \le v\}}$

- $\hat{G}^{\Delta}_{n,n}(v) = \frac{1}{T_n} \sum_{t=1}^{T} 1\{n_t = n,\ b^{n_t, n_t} + \Delta \le v\}$

- $\hat{F}_u(v) = \min_{\substack{n \in \{2,\dots,m\} \\ i \in \{1,\dots,n\}}} \varphi(\hat{G}_{i,n}(v), \underbrace{i}_{i^{th}\text{ order statistic}}, n)$

- $\hat{F}_L(v) = \max_{n \in \{2,\dots,m\}} \varphi(\hat{G}^{\Delta}_{n,n}(v), n-1, n)$

These are biased

Let $\hat{Y}_n = \varphi(\hat{G}_{n,n}; n-1, n)$

Take $\tilde{\hat{F}}_u = \sum_{n=2}^{m} \hat{Y}_n \frac{\exp(\hat{Y}_n \rho_T)}{\sum_{k=2}^{m} \exp(\hat{Y}_k \rho_T)}$

- as $\rho_T \to \infty$, $\tilde{F}_u(v) \to \hat{F}_u(v)$

$\hat{\pi}_L(r) = (r - v_0)(1 - \hat{F}_u(r))$

$\hat{\pi}_u(r) = (r - v_0)(1 - \hat{F}_L(r))$

Observable heterogeneity

- $\hat{G}_{i,n}(v|x) = \frac{\sum_t 1\{n_t = n;\ b^{i,n_t} \le v\} K_h(x - x_t)}{\sum_t K_h(x - x_t)}$

- in practice, it is good to smooth out those indicator functions

Confidence Intervals for Bounds

Want to estimate the set $\Theta = [\theta_l, \theta_u]$

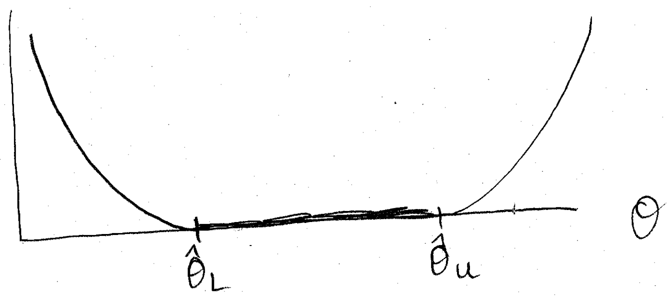
- ie know that $\theta_0 \in \Theta$

Want an estimate $\hat{\Theta} = [\hat{\theta}_l, \hat{\theta}_u]$

assume $r_n \begin{pmatrix} \hat{\theta}_l - \theta_l \\ \hat{\theta}_u - \theta_u \end{pmatrix} \xrightarrow{d} N(0, \Sigma)$, $\Sigma = \begin{bmatrix} \sigma_{uu} & \sigma_{ul} \\ \sigma_{ul} & \sigma_{ll} \end{bmatrix}$

CHT (Chernozhukov, Hong, Tamer):

- $\hat{\Theta} = \{\theta : Q_n(\theta) \le c\}$
- Take $Q(\theta) = [(\hat{\theta}_l - \theta)_+]^2 + [(\hat{\theta}_u - \theta)_-]^2$



Thus, here, we have: $[\hat{\theta}_L, \hat{\theta}_u] = \{\theta : Q_n(\theta) \le 0\}$

CHT confidence interval: $\mathcal{C} = \{\theta : Q_n(\theta) \le c'\}$

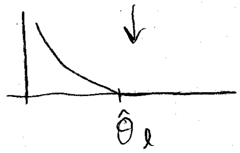
- $\mathcal{C} = [\hat{\theta}_l - \sqrt{c'}, \hat{\theta}_u + \sqrt{c'}]$

Want $\Pr[\Theta \subseteq \mathcal{C}] \to 1-\alpha$ (take this as a primitive)

- How do you pick c' satisfying these?

$\Theta \subseteq \mathcal{C}$ iff $q = \sup_{\theta \in \Theta} \hat{Q}_n(\theta) \le c'$

- $Q_n(\theta) = \max\{(\hat{\theta}_\ell - \theta)_+^2, (\hat{\theta}_u - \theta)_-^2\}$



$$\hat{q} = \sup_{\theta \in \Theta} Q_n(\theta) = \max\{(\hat{\theta}_\ell - \theta_\ell)_+^2, (\hat{\theta}_u - \theta_u)_-^2\}$$

$$r_n^2 \hat{q} = \max\{(r_n(\hat{\theta}_\ell - \theta_\ell)_+)^2, (r_n(\hat{\theta}_u - \theta_u)_-)^2\}$$

- This distribution exists by the continuous mapping theorem.
- simulate $w_\ell, w_u \sim N(0, \hat{\Sigma})$
- $\hat{d}$ is the $1-\alpha$ quantile of $\max\{(w_\ell)_+^2, (w_u)_-^2\}$

Then $\hat{\Theta} = \left[\hat{\theta}_\ell - \sqrt{\frac{\hat{d}}{r_n^2}}, \hat{\theta}_u + \sqrt{\frac{\hat{d}}{r_n^2}}\right]$.