More BLP

$S_j(p, x, \xi, \theta)$ ◦ need to solve for this numerically

$$\hat{S}_j^B(p, \delta, x, \alpha, \sigma) = \frac{1}{B}\sum_{b=1}^{B} \frac{\exp\{\alpha \ln(y_b - p_j) + \delta_j + \sum_{k=1}^{K} x_{jk}\sigma_k v_{kb}\}}{\exp\{\alpha \ln y_b + \delta_0 + \sigma v_{0b}\} + \sum_{l=1}^{J} \exp\{\alpha \ln(y_b - p_l) + \delta_l + \sum_{k=1}^{K} x_{lk}\sigma_k v_{kb}\}}$$

◦ only draw these simulations once

Solve $\hat{S}_j^B(p, \delta, x, \alpha, \sigma) = \underbrace{\hat{S}_j}_{\text{observed shares}}$ for the δ fcns:

$\hat{\delta}_j^B(\hat{S}, p, x, \alpha, \sigma)$.

Residuals are: $\hat{\delta}_j^B(\hat{s}, p, x, \alpha, \sigma) - x_j'\beta$, $j = 1, \ldots, J$

Let $\hat{g}(\theta) = \sum_{j=0}^{J} Z_j[\hat{\delta}_j^B(\hat{s}, p, x, \alpha, \sigma) - X_j'\beta]$

What are the Z_j's?

BLP: Z_j observable characteristics X, and cost shifters w (observable cost characteristics)

Hausman: prices in other markets

Critical assumption is that ξ_{jm} and p_{jm} are independent.

Optimal instruments

Recall: $E[\rho_j(\theta_0) \mid X_j, W_j] = 0$. In this model, the optimal instruments are:

$$E\left[\frac{\partial \rho_j}{\partial \theta}(\theta_0) \mid X, W\right] \left(E[\rho_j(\theta_0)^2 \mid X, W]\right)^{-1}$$

- assume homoskedasticity. If assume a symmetric NE,
  - Let $\mathcal{J}_j$ be the set of products produced by firm that produces product j.
  - $E\left[\frac{\partial \rho_j}{\partial \theta}(\theta_0) \mid X, W\right]$ depends only on:

$$X_j, W_j, \sum_{\substack{r \neq j \\ r \in \mathcal{J}_j}} X_r, \sum_{\substack{r \neq j \\ r \in \mathcal{J}_j}} W_r, \sum_{\substack{r \neq j \\ r \notin \mathcal{J}_j}} X_r, \sum_{\substack{r \neq j \\ r \notin \mathcal{J}_j}} W_r$$

  - This is already overidentified.

(*) Look at bias in Nevo with all his overidentifying assumptions.

$$\hat{\theta} = \text{argmin } g(\theta)' \hat{W} g(\theta)$$

- did not work very well.

add a supply side:

(*) Does Nevo use a supply side?

$$\ln mc_j = W_j' \gamma + \omega_j$$

$$\pi_f = \sum_{j \in \mathcal{J}_f} (p_j - mc_j) M s_j(p, x, \xi, \theta)$$

$\mathcal{J}_f$ - prods firm f produces.

FOCs:

$$(p_j): \; s_j(p,x,\xi,\theta) + \sum_{r \in \mathcal{J}_f} (p_r - mc_r) \frac{\partial s_r}{\partial p_j}(p,x,\xi,\theta) = 0$$

Let $\Delta_{jr} = \begin{cases} -\frac{\partial s_j}{\partial p_r} & r, j \in \mathcal{J}_f \text{ for some } f \\ 0 & \text{else} \end{cases}$

$$\Rightarrow \; s(p,x,\xi,\theta) - \Delta(p,x,\xi,\theta)\cdot(p - mc) = 0$$

$$\Rightarrow \; p = mc + \Delta^{-1}(p,x,\xi,\theta)\cdot s(p,x,\xi,\theta)$$

Let $b(p,x,\xi,\theta) = \Delta^{-1}(p,x,\xi,\theta)\, s(p,x,\xi,\theta)$

$$\Rightarrow \; mc = p - b(p,x,\xi,\theta)$$

$$\Rightarrow \; \underbrace{\text{"}\ln\text{"}}_{\text{the vector of logs.}}(p - b(p,x,\xi,\theta)) = w'\gamma + \omega$$

Thus, $\hat{g}(\theta) = \begin{bmatrix} \sum_{j=1}^{J} z_j \rho_j(\theta) \\ \sum_{j=1}^{J} \tilde{z}_j [\log(p_j - b_j(p,x,\xi,\theta)) - w_j'\gamma] \end{bmatrix}$

$$u_{ij} = \alpha \ln(y_i - p_j) + X_j'\beta + \xi_j + \varepsilon_{ij} + \underbrace{\sum_{k=1}^{K} X_{jk}\sigma_k \nu_k}_{\text{This is how this model differs from standard logit model}}$$

- does this allow enough correlation across goods?

Haile - Tamer (2002):

Is the reserve price in forest timber auctions set too low?

There is no variation in these reserve prices
- cannot use diffs-in-diffs

Can estimate the distribution of payoff functions, however

Two basic assumptions

1] Bidders don't bid more than they are willing to pay

2] Bidders don't let an opponent win with a price they are willing to beat.

$M \in \{2, \dots, \bar{m}\}$ potential bidders

$n \leq m$ actually bid

all bids observed

v_i, valuation of bidder i

b_i, bid for bidder i

Let $v^{(j:n)}$ be the j^{th} order statistic of v.
$b^{(j:n)}$ be the j^{th} order statistic of b

assumption 1 $\Rightarrow b^{(j:n)} \leq v^{(j:n)}$

Pf: Spse $v^{(j:n)} < b^{(j:n)}$. Then there are

$n-j+1$ bids above $v^{(j:n)}$, but only at most $n-j$ valuations above $v^{(j:n)}$. Thus, someone must be bidding more than their valuation. $\square$

Let Δ be the bid increment.

<u>Assumption 2</u> $\Rightarrow v^{(n-1:n)} \leq b^{(n:n)} + \Delta$

If $X \leq Y$, then $Y \leq r \Rightarrow X \leq r$

Thus $\{Y \leq r\} \subseteq \{X \leq r\}$

$$\Rightarrow F_Y(r) \leq F_X(r)$$

$$\boxed{1} \Rightarrow F_{v^{(j:n)}}(v) \leq F_{b^{(j:n)}}(v)$$

$$\boxed{2} \Rightarrow F_{b^{(n:n)}+\Delta}(v) \leq F_{v^{(n-1:n)}}(v)$$

<u>Example:</u>

$$F_{b^{(n:n)}+\Delta}(v) \leq F_{v^{(n-1,n)}}(v) \leq F_{b^{(n-1,n)}}(v)$$

Assume $v_1, \dots, v_n$ are iid from $F_0(v)$. Then,

$$F_{(j:n)}(v) = \frac{n!}{(n-j)!(j-1)!} \int_0^{F_0(v)} t^{j-1}(1-t)^{n-j}\,dt$$

Let $\varphi(F, j, n)$ solve $F = \frac{n!}{(n-j)!(j-1)!} \int_0^{\varphi(F,j,n)} t^{j-1}(1-t)^{n-j}\,dt$

i.e. it is an inverse function

Thus, we get a bound:

- $\varphi(F_{b(n:n)+\Delta}(v), n-1, n) \leq F(v)$

$$\leq \min_j \varphi(F_{b(j:n)}, j, n)$$

- This follows from applying the inverse transformation to:

$$F_{v(j:n)}(v) \leq F_{b(j:n)}(v)$$

and $F_{b(n:n)+\Delta}(v) \leq F_{v(n-1:n)}(v)$

This can give us bounds on optimal reserve prices and revenues.

Taking the minimum introduces a pretty big amount of bias.