

## Basic hold-up problem

1 buyer, 1 seller

◦ value to buyer  $v(q)$ ,  $v' > 0$ ,  $v'' < 0$

◦ cost to seller  $F(i)$ ,  $F' > 0$ ,  $F'' > 0$

◦  $q$  - investment by buyer (at cost  $q$ )

◦  $i$  - investment by seller (at cost  $i$ )

◦ assume they cannot write a contract on  $(q, i)$

◦ observable but not verifiable

Stage 1: B, S choose  $q, i$  respectively

Stage 2:  $q, i$  observed, buyer and seller

negotiate price  $p$ .

Assumption: B, S have equal bargaining power

◦ split surplus evenly.

◦ each receive  $\frac{v-F}{2}$ .

Stage 2: Price is  $p = \frac{v+F}{2}$

Stage 1 investment decisions:

$$(B): \max_q \frac{v(q) - F(i)}{2} - q$$

$$(q): \frac{v'(q)}{2} - 1 = 0 \Rightarrow v'(q^*) = 2$$

$$(S): \max_i \frac{v(q) - F(i)}{2} - i$$

$$(i): -F'(i^*) = 2$$

First best:  $\max q, i \quad v(q) - F(i) - q - i$

$$(q): v'(q^{FB}) = 1$$

$$(i): -F'(i^{FB}) = 1$$

Since  $v$  concave  $q^{FB} > q^*$   
 $F$  convex  $i^{FB} > i^*$

## Hart- Tirole

### Vertical Integration and Market Foreclosure

(\*) Read Ordover - Salop - Saloner

Basic model:

- Two upstream firms  $u_1, u_2$  sell homogeneous good at marginal costs  $c_1 < c_2$  resp.
- Two downstream firms  $D_1, D_2$  use input, produce final good, which is perf. sub. Compete Bertrand.

Ex post monopolization

- relatively efficient upstream firm merges with downstream firm to restrict output in the downstream market.
- cost  $e$  of merging
- assume initially that  $c_2 = +\infty$

◦ Imagine  $U_1$  is selling  $q^m$  to  $D_1$ .

Recall: 2 firms, 2 stages. In stage 1, choose capacities. Stage 2 compete Bertrand. This is equivalent to Cournot competition.

◦ So that  $U_1$  doesn't have the incentive to sell to  $D_2$ , it may be profitable for  $U_1$  and  $D_1$  to merge.

### Scarce Needs

◦ Assume  $c_1 = c_2$

◦  $D_1, D_2$  have limited needs for the good produced from  $U_1$  and  $U_2$ .

◦ Assume firms bargain over gains from trade and upstream firms receive a positive share

◦  $U_1$  may have incentive to merge with  $D_1$  to ensure that  $D_1$  does not purchase from  $U_2$ .

◦ gains an additional trading opportunity at the expense of  $U_2$ .

◦ can lead to exit of  $U_2$ , leading to monopolization

### Variant 3: Scarce supplies

- scarce resource upstream

◦  $q < q^m$   
upstream output      downstream monopoly quantity

- same incentive for merging as in scarce needs setup.

