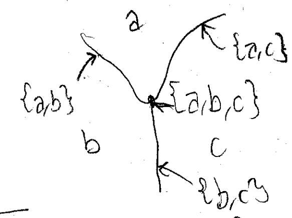
Structure of Rationalizability: applications



Robustness of predictions

- Which type of refinements are robust?

Model: $M = (N, A, u, \Theta \times T, p)$

Solution concept: $\Sigma : M \mapsto \Sigma(M) \subseteq \Delta(\underbrace{A^T}_{\text{strategy profiles}})$

Non-emptiness: Σ is nonempty iff $\Sigma(M) \neq \emptyset$ for each finite M with CPA

Refinements: Σ is a refinement of rationalizability iff $s(t) \in S^\infty[t]$ $\forall t \in T$ $\forall s \in supp(\sigma)$ and $\forall \sigma \in \Sigma(M)$

Predictions: a formula Q with free variable $s: T \to A$ is a prediction (Σ, M) iff

$$Q(s) \text{ is true } \forall s \in supp(\sigma), \forall \sigma \in \Sigma(M)$$

- This is the stuff that is true under a solution

Metric on T^*: Let d be consistent with the product topology on T^*.

Perturbation: (T', τ) is (ε, k)-perturbation of M if

- $T' \subseteq T^*$ is finite, belief-closed, and has a common prior. (T' is a "nice" model)
- $\tau: T \to T'$ s.t. $\forall t$, $\forall l \leq k$, $d(\tau^l(t), t^l) \leq \varepsilon$.

Robustness: Q is (ε, k) robust if $\forall (\varepsilon, k)$ perturbation (T, τ), $\forall \sigma \in \Sigma(T)$, $\forall s \in \text{supp}(\sigma)$, $Q(s \circ \tau)$ is true.

Q is robust iff it is (ε, k)-robust for some $\varepsilon > 0$ and $k < \infty$

Thm: $\forall$ nonempty refinement Σ of rationalizability and any finite model M, a prediction Q of (Σ, M) is robust iff Q is a robust prediction of (S^∞, M)

($\Rightarrow$) Follows from upper-hemicontinuity of rationalizability.

($\Leftarrow$) Follows from definitions

	A	NA
A	θ, θ	$\theta-1, 0$
NA	$0, \theta-1$	$0, 0$

$\theta = \frac{2}{3}$: Rubinstein $\Rightarrow$ A is not robust

CVD, K-M $\Rightarrow$ NA is "robust"

example earlier $\Rightarrow$ NA is not robust

Consider $(N, \Omega, I, \underbrace{\mu}_{\text{common prior}})$

want to define a notion of approximate common knowledge under which solution concepts are continuous.

Define $B_i^p(E) = \{\omega : \mu(E \mid I_i(\omega)) \geq p\}$ p-beliefs

Lemma: $B_i^p(E) \in \underbrace{\mathcal{K}_i}_{\text{knowledge field}}$ since this is a property of $I_i(\omega)$, not ω itself.

- $E \in \mathcal{K}_i \underset{\uparrow}{\Rightarrow} B_i^p(E) = E$, $p > 0$.

  know that you know it

- $E \subseteq F \underset{\uparrow}{\Rightarrow} B_i^p(E) \subseteq B_i^p(F)$ since μ is monotone.

  logical omniscience

- Consider $E^0 \supseteq E^1 \supseteq E^2 \supseteq \ldots$. Then,

$$B_i^p\left(\bigcap_{n=1}^{\infty} E^n\right) = \bigcap_{n=1}^{\infty} B_i^p(E^n)$$

- $\mu(E \mid B_i^p(E)) \geq p$

<u>Defn</u>: E is <u>p-evident</u> iff $\forall i$, $E \subseteq B_i^p(E)$ (everything is $p=0$-evident)
(cf: E is evident iff $\forall i$, $E \subseteq K_i(E)$)

Suppose $\theta \in \{0,1\}$, and suppose an auctioneer announces price that each person hears with probability $1-p$.

s_i \ θ	0	1
0	p	$1-p$
1	$1-p$	p

- $E = \{(\overset{\theta_i=0}{0}, \overset{s_1=0}{0}, \overset{s_2=0}{0})\}$
- $\Pr[\theta = 0 \mid s_i = 0] = \frac{p}{p+(1-p)} = p$
- $\Pr[\theta = 0 \mid s_i = 1] = 1-p$

- $B_i^{p^2}(E) = \{s_i = 0\} \supseteq E$

Let $E = \{\theta = 0\}$. Is this p^2 evident?
$B_i^{p^2}(E) = \{s_i = 0\} \not\supseteq E$.

If $\pi > 1-p$, then E cannot be π evident

Let $x_i = \theta + \eta_i$ $\eta_i \sim U[-\varepsilon, \varepsilon]$

- can we have p-evidence for $p \geq \frac{1}{2}$?
- what do p-evident events look like here?

Defn: C is a common p-belief at ω iff $\exists E \ni \omega$ s.t.

i) E is p-evident

ii) $E \subseteq B_i^p(C)$ $\forall i$.

cf: C is common knowledge at ω iff $\exists E \ni \omega$ s.t.

i) E is evident (public)

ii) $E \subseteq C$

- $E = K_i(E) \subseteq K_i(C)$

Defn: Let $E^p(C) = \bigcap_{n \geq 1} C^n$, where

- $C^0 = C$
- $C^n = \bigcap_{i=1}^{n} B_i^p(C^{n-1})$

Proposition: $\forall C, p$,

1. $E^p(C)$ is p-evident, and $E^p(C) \subseteq B_i^p(C)$ $\forall i$
   - whenever C occurs, everyone assigns at least probability p to it
2. C is common p-belief at ω iff $\omega \in E^p(C)$

1] Observe $C^{n+1} \subseteq C^n$ since $C^n \subseteq B_i^p(C^{n-1})$

$\Rightarrow B_i^p(C^n) \subseteq B_i^p(B_i^p(C^{n-1})) = B_i^p(C^{n-1})$

$\Rightarrow C^{n+1} = \bigcap_i B_i^p(C^n) \subseteq \bigcap_i B_i^p(C^{n-1}) = C^n$

Next $E^p(C) \subseteq C^{n+1} \subseteq B_i^p(C^n)$

$\Rightarrow E^p(C) \subseteq \bigcap_{n \geq 1} B_i^p(C^n)$

Then, $E^p(C) \subseteq \bigcap_{n \geq 1} B_i^p(C^n) = B_i^p(\bigcap_{n \geq 1} C^n) = B_i^p(E^p(C))$

$\Rightarrow E^p(C)$ is a p-evident event.

For 2], $(\Leftarrow)$ direction is just 1].

- $B_i^p(E) \supseteq E$
- $B_i^p(\bigcap_i B_i^p(E)) \supseteq E \quad \Rightarrow E \subseteq E^p(C)$

If common p-belief that posteriors are $(\pi_1, \dots, \pi_n)$, then

$|\pi_i - \pi_j| \leq 2(1-p)$

- This generalizes the Agreeing to Disagree thm.