<u>Defn</u>: $(C, \Theta, f, (t_i)_{i \in N})$ has no redundant types if h is 1-1.

<u>Thm</u>: If $(C, \Theta, f, (t_i)_{i \in N})$ has no redundant types, then h is an isomorphism (h and h^{-1} is continuous).

Embedding in T^*-morphisms

<u>Defn</u>: a <u>belief morphism</u> from $(C, \Theta, f, (t_i)_{i \in N})$ to $(\underline{C}, \underline{\Theta}, \underline{f}, (\underline{t}_i)_{i \in N})$ is a pair of continuous $\varphi: \Theta \to \underline{\Theta}$ and $\varphi': C \to \underline{C}$ s.t. the following diagram commutes:

$$\begin{tikzcd}
\Theta \arrow[r, "\varphi"] & \underline{\Theta} \\
C \arrow[u, "f"] \arrow[r, "\varphi'"] \arrow[d, "t"'] & \underline{C} \arrow[u, "\underline{f}"'] \arrow[d, "\underline{t}"'] \\
(\Delta(C))^n \arrow[r, "(\varphi')^*"'] & (\Delta(\underline{C}))^n
\end{tikzcd}$$

i.e. if $\underline{f}(\varphi'(c)) = \underline{f}(\underline{c}) = \underline{\theta}$

$\Rightarrow \varphi(f(c)) = \varphi(\theta) = \underline{\theta}$

If $(\underline{C}, \underline{\Theta}, \underline{f}, (\underline{t}_i)_{i \in N})$ has no redundant types,

something wrong here $\{$ $\varphi' = \underline{h}^{-1} \circ h \circ \varphi$, where $\underline{h}$ is the hierarchy for $(\underline{C}, \underline{\Theta}, \underline{f}, (\underline{t}_i)_{i \in N})$

Defn a belief morphism (φ, φ') is a belief isomorphism if $(\varphi^{-1}, \varphi'^{-1})$ exists and is a belief morphism.

$$\begin{tikzcd}
\Theta & \underline{\Theta} \arrow[l, "\varphi^{-1}"'] \\
C \arrow[u, "f"] \arrow[d, "t"'] & \underline{C} \arrow[l, "\varphi'^{-1}"'] \arrow[u, "\underline{f}"'] \arrow[d, "\underline{t}"] \\
(\Delta(C))^n & (\Delta(\underline{C}))^n \arrow[l, "((\varphi')^{-1})^*"]
\end{tikzcd}$$

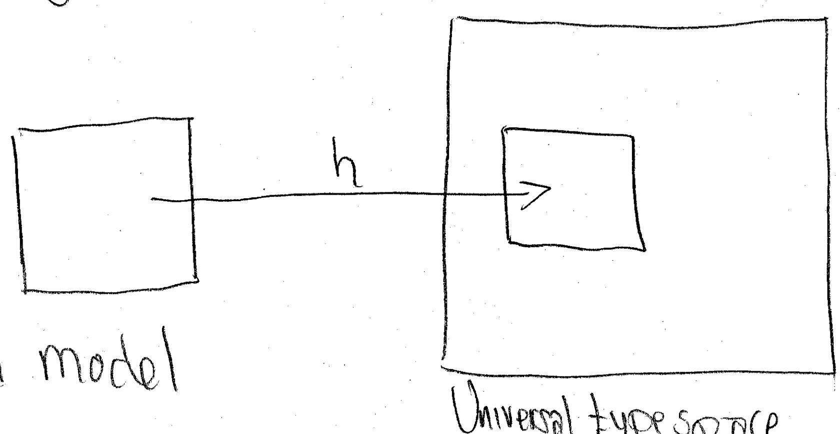
Thm: For any $(C, \Theta, f, (t_i)_{i\in N})$, (identity, h) is a belief morphism from $(C, \Theta, f, (t_i)_{i\in N})$ to the universal type space $(\Theta \times T^*, \Theta, Proj_\Theta, \kappa)$

$$\begin{tikzcd}
\Theta \arrow[r, "i"] & \Theta \\
C \arrow[u, "f"] \arrow[r, "h"] \arrow[d] & \Theta \times T^* \arrow[u, "Proj_\Theta"'] \arrow[d] \\
(\Delta(C))^n \arrow[r, "h^*"] & (\Delta(\Theta \times T^*))^n
\end{tikzcd}$$

Pf: h is continuous, and $f(c) = h^\circ(c)$. $\square$

Thm: If $(C, \Theta, f, (t_i)_{i \in N})$ has no redundant types, then (identity, h) is a belief isomorphism from $(C, \Theta, f, (t_i)_{i \in N})$ to $(\Theta \times T^*, \Theta, Proj_\Theta, \kappa)$

Embedding in T^*



$$c_n \to c \Rightarrow h(c_n) \to h(c)$$

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a subset T of T^* is a <u>belief-closed subspace</u> if for each t_i, $supp(\kappa(t_i)) \subseteq \Theta \times T_{-i}$

For any model $(\Theta \times T, p_1, p_2, \ldots, p_n)$, $h(T)$ is a belief-closed subspace

a type profile t in T^* is <u>finite</u> if $t = h(t')$ for some finite model $(\Theta' \times T', p_1, \ldots, p_n)$, where $|\Theta' \times T| < +\infty$

Thm (Mertens and Zamir): The set of finite types is dense: for every $t_i \in T_i^*$, $\exists$ a sequence of finite types $t_{i,m}$ with $t_{i,m} \to t_i$ as $m \to \infty$.

Lipman's paper on the common prior assumption

The set of models with common priors must be much smaller than the entire set of models, one would imagine.

- No. The set of models with finite types and common prior is dense.

Lipman: For any finite model where beliefs have full support and for any K, $\exists$ a model with common priors such that $\forall t$ in the original model, $\exists$ a type in the new model whose first K orders of beliefs are same as of t

If you have a result that depends only on finite-order beliefs, then the result does not depend on the common prior assumption.

Recall: $h^k(t)$ is the profile of k^{th}-order belief hierarchies for t!

any finite model
$M = (\Theta \times T_1 \times \ldots \times T_n, p_1, \ldots, p_n)$, where p_i has full support.

For any k, $\exists$ a finite model with common priors
$M^{CPA} = (\Theta \times \underline{I}_1 \times \ldots \times \underline{I}_n, p)$ and a 1-1 mapping, $\tau : T \to \underline{I}$ s.t. $\forall t \in T$,
$h^k(\tau(t)) = h^k(t)$.

Pf: Take $n=2$ for simplicity