	I	NI
I	θ, θ	$\theta-1, 0$
NI	$0, \theta-1$	$0, 0$

θ is not common knowledge

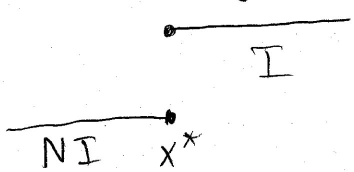
$x_i = \theta + \varepsilon \eta_i$

- This game is monotone supermodular

$$F(\theta, x_j | x_i) = Pr[-\varepsilon \eta_i < \theta - x_i, \varepsilon(\eta_j - \eta_i) < x_j - x_i | x_i]$$

- decreasing in x_i.

Cut-off strategies: NI if $x_i < x^*$, I if $x_i > x^*$



Invest $\Leftrightarrow$ $x_i \geq Pr[x_j < x^* | x_i]$

Cut-off value: $x^* = Pr[x_j < x^* | x^*] = \frac{1}{2}$ by symmetry

What is special about this $x^* = \frac{1}{2}$?

Risk dominance: In a 2x2 game, a strategy is "risk dominant" iff it is a BR when the other player plays each strategy with equal probabilities.

	I	NI
I	θ, θ	$\theta-1, 0$
NI	$0, \theta-1$	$0, 0$

Invest is RD iff

$$0.5\,\theta + 0.5(\theta - 1) > 0$$

$$\Leftrightarrow \theta > \frac{1}{2}$$

- In this game, players play the risk dominant strategy.

"How many 2×2 symmetric games are there?"
"17." Jason
"Huh?"
"Well, there are a continuum, but 17 sounds like a good answer." Me
"Actually, there are three, depending how you count"

Carlsson and Van Damme

Risk Dominance:

(g)

	A	B
A	u_{11}, v_{11}	u_{12}, v_{12}
B	u_{21}, v_{21}	u_{22}, v_{22}

- suppose (A,A) and (B,B) are NE
- (A,A) is risk dominant if

$$(u_{11}-u_{21})(v_{11}-v_{12}) > (u_{22}-u_{12})(v_{22}-v_{21})$$

u_{11}	u_{12}
u_{21}	u_{22}

$\Longleftrightarrow$

u_{11}	$u_{12}-u_{12}$
u_{21}	$u_{22}-u_{12}$

- player 1's payoffs are displayed

$\Longleftrightarrow$

$u_{11}-u_{21}$	0
0	$u_{22}-u_{12}$

- These games are strategically equivalent under standard solution concepts

- Turn (g) into (g^a)

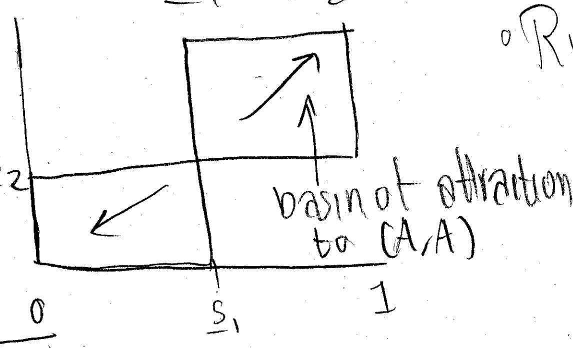
(g^a):

	A	B
A	g_1^a, g_2^a	0, 0
B	0, 0	g_1^b, g_2^b

Here, (A, A) is risk dominant iff $a^2 > 1$

- Pareto dominance is not robust to these types of transformations.

I is indifferent against $\underline{s}_i$. (A, A) is risk-dominant iff $\underline{s}_1 + \underline{s}_2 < 1$ $\qquad \underbrace{\underline{s}_j}_{Pr[j \text{ plays } A]}$



- Risk dominance: which equilibrium has the bigger basin of attraction?

Dominance Region:

- $D_i^a = \{(u,v) \mid g_i^b < 0, g_i^a > 0\}$
  $= \{\text{games in which } a \text{ is dominant for } a\}$

Risk-dominance region:

- $R^a = \{(u,v) \mid g_1^a > 0, g_2^a > 0, \text{ and if } g_1^b, g_2^b > 0, \text{ then } \underline{s}_1 + \underline{s}_2 < 1\}$
  $= \{(u,v) \mid g_1^a g_2^a > g_1^b g_2^b, g_1^a, g_2^a > 0\}$

Clearly, $R^a \subseteq D^a = D_1^a \cap D_2^a$

assume:

- $\Theta \subseteq \mathbb{R}^m$ is open; (u,v) are continuously differentiable functions of θ with bounded derivatives.
  - e.g. In our original example, $\theta \overset{u}{\mapsto} (\theta, \theta-1, 0, 0)$
- The prior on θ has a density h which is strictly positive, continuously differentiable, and bounded;
  - support is Θ
- Each player observes $x_i = \theta + \varepsilon \eta_i$
  - (η_1, η_2) is bounded
  - Independent of θ
  - admits a continuous density.

Thm: (Risk dominance vs. rationalizability)

assume
- x is on a continuous curve $C \subseteq \Theta$, where
- $(u(c), v(c)) \in R^a$ for each $c \in C$
- $(u(c), v(c)) \in D^a$ for some $c \in C$

Then, A is the only rationalizable action at x when ε is small.