I. Global games with common vs. private values
II. Global games without dominance regions.
III. Dynamic global games.

(G1)

1 \ 2	I	NI
I	θ, θ	$\theta-1, 0$
NI	$0, \theta-1$	$0, 0$

$x_i = \theta + \varepsilon \eta_i$, $i \in \{1,2\}$

$(\eta_1, \eta_2) \perp\!\!\!\perp \theta$, $\eta_1 \perp\!\!\!\perp \eta_2$

- $\theta \sim \underbrace{f_\theta}_{\text{prior}}$ with convex support
- $\eta_i \sim f_\eta$

Dominance regions:

$Pr[\theta > 1] > 0$
$Pr[\theta < 0] < 0$
} positive measure of dominance regions

- If $\theta > 1$, I is dominant
- If $\theta < 0$, NI is dominant.

(G1) is a common value global game

(G2)

1 \ 2	I	NI
I	x_i, x_{-i}	$x_i - 1, 0$
NI	$0, x_{-i} - 1$	$0, 0$

$x_i = \theta + \varepsilon \eta_i$
$\theta \sim f_\theta$
$\eta_i \sim f_\eta$

- you get your signal, but you are trying to learn the other player's signal, which is correlated with your own due to θ.

as $\varepsilon \to 0$, (G1) and (G2) have similar properties.

<u>CVD1:</u> With dominance regions, $\exists\ \bar{\varepsilon} > 0$ s.t. $\forall \varepsilon < \bar{\varepsilon}$, G_ε has a unique threshold form equilibrium $(x^{*,1}_\varepsilon, x^{*,2}_\varepsilon)$

<u>CVD2:</u> as $\varepsilon \to 0$, $x^{*,1}_\varepsilon \to x^{rd}$, where x^{rd} is the risk dominance threshold.

Risk dominance:

π_i	I	NI
I	a_i	b_i
NI	c_i	d_i

When (I,I) and (NI, NI) are both equilibria, with $a_i \geq c_i, d_i \geq b_i$ for $i \in \{1,2\}$, we say that (I,I) is <u>risk dominant</u> iff $\prod_i (a_i - c_i) \geq \prod_i (d_i - b_i)$.

- Frankel, Morris, and Shin (JET '03),
- Carlsson and van Damme (Ecta '93),
- Harsanyi and Selten (Book, '88)

Here, x^{rd} = value of θ s.t. $\underbrace{(\theta)^2}_{(a_1-c_1)(a_2-c_2) = (d_1-b_1)(d_2-b_2)} = (1-\theta)^2$

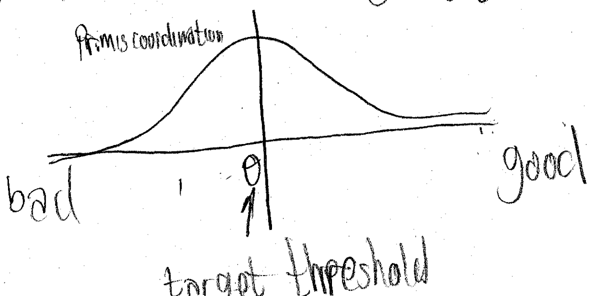
where $a_1 = \theta, c_1 = 0, b_1 = \theta - 1, d_1 = 0$

$\Rightarrow \theta = \frac{1}{2}$

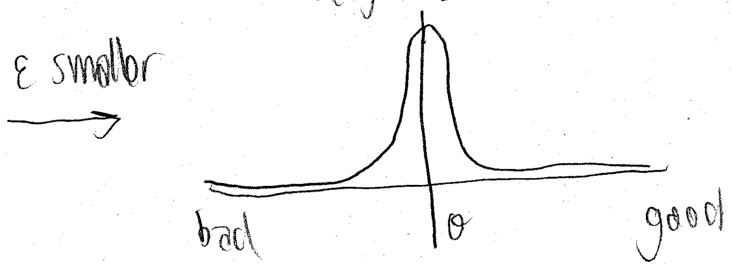
We want to introduce the probability of miscoordination.

Two ways:

1. Probability q of mistake
2. Different information creates mis coordination.



- people don't mis-coordinate when things are good or when things are bad. They miscoordinate when things are noisy



- The height of this peak does not shrink.

Private value game:

	I	NI
I	x_i, x_i	$x_i - 1, 0$
NI	$0, x_{-i} - 1$	$0, 0$

$x_i = \theta + \varepsilon \eta_i$

$\theta \sim f_\theta$, $\eta_i \sim f_\eta$

$\eta_i \perp\!\!\!\perp \eta_j$

1. Existence of extreme, threshold-form equilibrium
   - $\underline{x}_\varepsilon$, $\overline{x}_\varepsilon$, $\text{supp}(\theta) = \mathbb{R}$

Could have: $\underline{x}_\varepsilon \in \{-\infty, +\infty\}$ or can have $\underline{x}_\varepsilon, \overline{x}_\varepsilon \in \mathbb{R}$

$\overline{x}_\varepsilon \in \{-\infty, +\infty\}$

Order: $s: x \longmapsto \{I, NI\}$
$\{1, 0\}$

$s' \preceq s$ iff $s'(x) \leq s(x) \quad \forall x \in \mathbb{R}$

- $s' \preceq s$ if s makes you invest more

$\underline{s}_\varepsilon = \begin{cases} I & \text{if } x \geq \underline{x}_\varepsilon \\ NI & \text{if } x < \underline{x}_\varepsilon \end{cases}$ $\quad \underline{s}_\varepsilon \preceq \bar{s}_\varepsilon$ iff $\underline{x}_\varepsilon \geq \bar{x}_\varepsilon$

Dominance Regions:

Upper dominance $(\theta \geq 1) \Rightarrow \underline{x}_\varepsilon \in \mathbb{R} \cup \{-\infty\}$
Lower dominance $(\theta < 0) \Rightarrow \bar{x}_\varepsilon \in \mathbb{R} \cup \{+\infty\}$

- If have both, $\underline{x}_\varepsilon, \bar{x}_\varepsilon \in \mathbb{R}$

If x^*_ε is a symmetric equilibrium threshold, then $E[u_I \mid x_i = x^*_\varepsilon] = E[u_{NI} \mid x = x^*_\varepsilon]$

- indifference condition
  - $s(x^-_\varepsilon) \to NI \qquad E[u_I \mid x^+_\varepsilon] \approx E[x_I \mid x^-_\varepsilon]$
  - $s(x^+_\varepsilon) \to I \qquad E[u_{NI} \mid x^+_\varepsilon] \approx E[x_{NI} \mid x^-_\varepsilon]$
    - by continuity of opponent's probabilities, and

x^*_ε NE threshold, $E[u_I \mid x^+_\varepsilon] \geq E[u_{NI} \mid x^+_\varepsilon]$
$E[u_{NI} \mid x^-_\varepsilon] \geq E[u_I \mid x^-_\varepsilon]$

	I	NI
I	x_i	$x_i - 1$
NI	0	0

$x_i = x_\varepsilon^*$

$$x_\varepsilon^* - \underbrace{\Pr[x_{-i} \le x_\varepsilon^* \mid x_i = x_\varepsilon^*]}_{=\frac{1}{2}} = 0$$

$$\Rightarrow x_\varepsilon^* = \tfrac{1}{2}$$

$$\Pr[x_{-i} \le x_\varepsilon^* \mid x_i = x_\varepsilon^*]$$

$$= \frac{\int_\theta f_\theta(\theta) f_\eta\left(\frac{x_i-\theta}{\varepsilon}\right) P_\eta\left(\eta_{-i} \le \frac{x_i-\theta}{\varepsilon}\right) d\theta}{\int_\theta f_\theta(\theta) f_\eta\left(\frac{x_i-\theta}{\varepsilon}\right) d\theta}$$

$$x_i = \theta + \varepsilon\eta \Leftrightarrow \eta = \frac{x_i-\theta}{\varepsilon}$$

Let $u = \frac{x_i-\theta}{\varepsilon}$. Then $d\theta = -\varepsilon du$

$$\Pr[x_{-i} \le x_\varepsilon^* \mid x_i = x_\varepsilon^*]$$

$$= \frac{\int_\theta f_\theta(x_i - \varepsilon u) f_\eta(u) P_\eta(\eta_{-i} \le u) du}{\int_\theta \underbrace{f_\theta(x_i - \varepsilon u)}_{\to f_\theta(x_i)} f_\eta(u) du}$$

$$\to \int f_\eta(u) P(\eta_{-i} \le u) du = \int \frac{\partial}{\partial u}\left(\tfrac{1}{2} P_\eta^2\right) du$$

$$= \tfrac{1}{2}\left[\underbrace{P_\eta^2(\eta_i < +\infty)}_{\to 1} - \underbrace{P_\eta^2(\eta_i < -\infty)}_{\to 0}\right] \to \tfrac{1}{2} \quad \square$$

Imagine lower dominance region but no upper dominance region.

NI always equilibrium

Most investment eq $\Rightarrow x^{rd}$