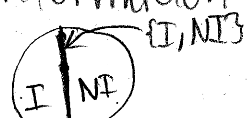
Global Games

- Theory
  - 2×2 games (Carlsson and Vamme)
  - Continuum of players (Morris and Shin)
    - each player has two actions
  - General supermodular games
- Applications
  - Bank runs
  - Currency attacks (Morris and Shin)

Two similar environments might have different outcomes
- multiple equilibria with strategic complementarity

	I	NI
I	θ,θ	$\theta-1,0$
NI	$0,\theta-1$	$0,0$

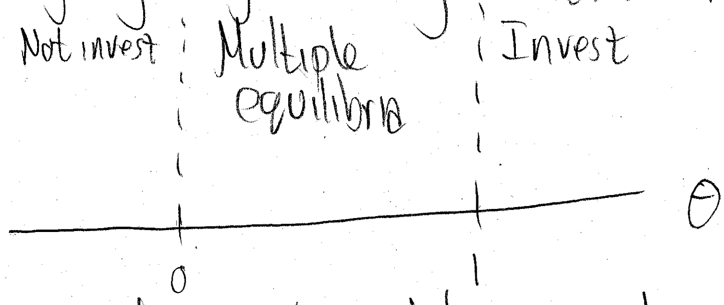
  - If $0<\theta<1$, there are two pure strategy equilibria.
  - investment game, search
- introducing incomplete information leads to dominant solvability.



- only along this line can there be multiple equilibria.

Typically assume θ is common knowledge. What if we relax this slightly by adding ε-amount of noise?

	I	NI
I	θ,θ	$\theta-1,0$
NI	$0,\theta-1$	$0,0$



- under common knowledge, this is how things look.

Suppose θ is <u>not</u> common knowledge
- θ is uniformly distributed over a large interval

- private signal: $x_i = \theta + \varepsilon\eta_i$, $\varepsilon > 0$ small and common knowledge
- (η_1, η_2) is bounded, $(\eta_1, \eta_2) \perp\!\!\!\perp \theta$
- η_i iid w/ continuous cdf F
- $E[\eta_i] = 0$

Conditional beliefs

$\theta \overset{d}{=} x_i - \varepsilon\eta_i$ this follows from $\theta \sim$ uniform

$$\Rightarrow \Pr[\theta \le \underline{\theta} \mid x_i] = \Pr[x_i - \varepsilon\eta_i \le \underline{\theta} \mid x_i]$$
$$= \Pr\left[\eta_i \ge \frac{x_i - \underline{\theta}}{\varepsilon} \,\Big|\, x_i\right] = 1 - F\left(\frac{x_i - \underline{\theta}}{\varepsilon}\right)$$

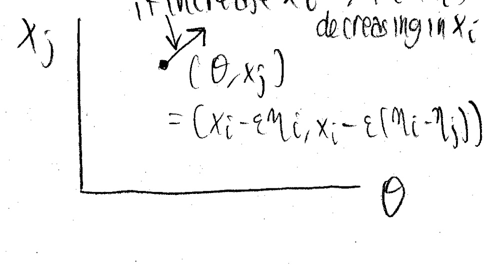
$\Rightarrow F_{\theta|x_i}(\theta) = 1 - F\left(\frac{x_i - \theta}{\varepsilon}\right)$ F is decreasing in x_i $\Rightarrow$ FOSD when $x_i < x_i'$

$$x_j \overset{d}{=} \theta + \varepsilon\eta_j \overset{d}{=} (x_i - \varepsilon\eta_i) + \varepsilon\eta_j \overset{d}{=} x_i - \varepsilon\eta_i + \varepsilon\eta_j$$

$$\Pr[x_j \le x \mid x_i] = \Pr[x_i - \varepsilon(\eta_i - \eta_j) \le x \mid x_i]$$
$$= \Pr[\varepsilon(\eta_j - \eta_i) \le x - x_i \mid x_i]$$
$$= \Pr\left[\eta_j - \eta_i \le \frac{x - x_i}{\varepsilon} \,\Big|\, x_i\right]$$

Let F_Δ be the CDF of $\eta_j - \eta_i$

Then $F_{x_j|x_i}(x) = F_\Delta\left(\frac{x - x_i}{\varepsilon}\right) \Rightarrow$ increasing in the sense of FOSD in x_i



Finally, $F(\theta, x_j \mid x_i)$ is decreasing in x_i

$$F(\theta, x_j \mid x_i + \Delta) = F(\theta - \Delta, x_j - \Delta \mid x_i)$$
$$< F(\theta, x_j \mid x_i)$$

and $E[\theta \mid x_i] = x_i$.

Suppose see signal x_i. Then player i's payoffs are:

$E[\theta \mid x_i]$	I	NI
I	x_i	$x_i - 1$
NI	0	0

Define the order $\geq$ on action set by $I \geq NI$. Then $U_i(a_i, a_j, x_i)$ is supermodular

$$\underbrace{U(I,I,x_i) - U(NI,I,x_i)}_{x_i - 0} \text{ vs } \underbrace{U(I,NI,x_i) - U(NI,NI,x_i)}_{x_i - 1}$$
$$>$$

$\Rightarrow$ increasing differences in (a_i, a_j)

Thus, this is a supermodular game, so there exist smallest and largest rationalizable strategies which are BNE and isotone.

$\Rightarrow$ if invest when $x_i = x$, must invest when $x_i = x' > x$.

∘ Must have cut-off strategies. (cut-off could be $+/-\infty$).

Compute these monotone BNE.

	I	NI
I	x_i	$x_i - 1$
NI	0	0

$$U(I,\cdot) = x_i \Pr[a_2 = I \mid x_i] + (x_i - 1)\Pr[a_2 = NI \mid x_i]$$
$$= x_i - \Pr[a_i = NI \mid x_i]$$
$$U(NI,\cdot) = 0$$

invest if $x_i \geq \Pr[a_i = NI \mid x_i]$

assume $\mathrm{supp}(\Theta) = [a,b]$, $a < 0 < 1 < b$

$x_i < 0 \Rightarrow s_i^*(x_i) = NI$ since $x_i - \Pr[x_i = NI \mid x_i] < 0$

$x_i > 1 \Rightarrow s_i^*(x_i) = I$ since $\underbrace{x_i}_{>1} - \underbrace{\Pr[x_i = NI \mid x_i]}_{\leq 1} \geq 0$

$\Rightarrow \exists x_i^* \in [0,1]$ s.t. $s_i^*(x_i) = \begin{cases} NI & x_i < x_i^* \\ I & x_i > x_i^* \end{cases}$

since payoffs are continuous

Indifference when $x_i = x_i^*$

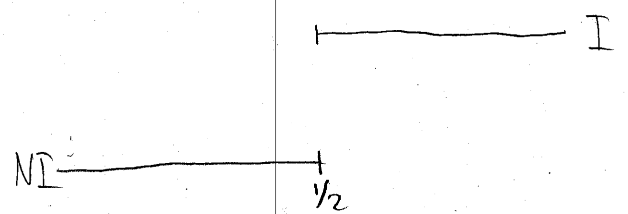
Symmetry: $x_1^* = x_2^* = x^*$

$\Rightarrow x^* = \Pr[x_j \le x^* \mid x^*] = F_\Delta\left(\frac{x^*-x^*}{\varepsilon}\right) = \frac{1}{2}$ by symmetry

$= F_\Delta(0) = \Pr[\eta_i < \eta_j \mid x^*]$

Thus, each player invests if $x_i > \frac{1}{2}$, not invests if $x_i < \frac{1}{2}$, and is indifferent if $x_i = \frac{1}{2}$.

What is the smallest BNE?



Smallest BNE:
NI iff $x_i \le \frac{1}{2}$

Largest BNE:
NI iff $x_i < \frac{1}{2}$

Which strategies are rationalizable? $\{\text{NI iff } x_i \le \frac{1}{2}\}$ and $\{\text{NI if } x_i < \frac{1}{2}\}$

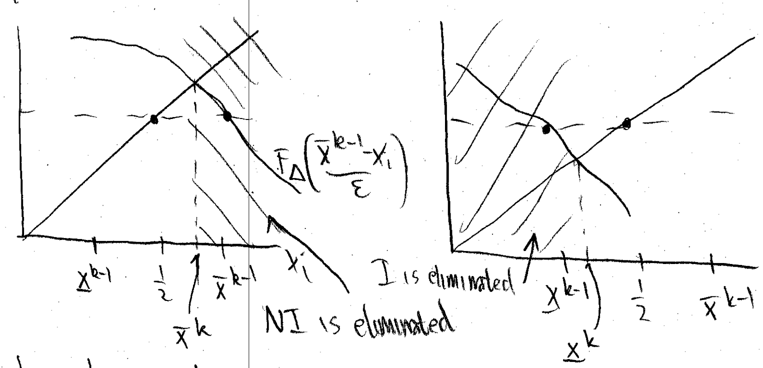
Let $\underline{x}^0 = 0$, $\bar{x}^0 = 1$. Suppose $x_i < \underline{x}^0 \Rightarrow$ I eliminated for i

Suppose $x_i > \bar{x}^0 \Rightarrow$ NI eliminated for i

Round k-1: $\exists\, \underline{x}^{k-1} < \frac{1}{2} < \bar{x}^{k-1}$ s.t. $x_i < \underline{x}^{k-1} \Rightarrow$ I eliminated for i

Suppose: $x_i > \bar{x}^{k-1} \Rightarrow$ NI eliminated for i

$F_{x_j|x_i}(\underline{x}^{k-1}) \le \Pr[s_j = NI \mid x_i] \le F_{x_j|x_i}(\bar{x}^{k-1}) = F_\Delta\left(\frac{\bar{x}^{k-1} - x_i}{\varepsilon}\right)$



$\bar{x}^k = F_\Delta\left(\frac{\bar{x}^{k-1} - \bar{x}^k}{\varepsilon}\right)$

$\Rightarrow \bar{x}^\infty = F_\Delta\left(\frac{0}{\varepsilon}\right) = \frac{1}{2}$

$\Rightarrow \underline{x}^\infty = F_\Delta\left(\frac{0}{\varepsilon}\right) = \frac{1}{2}$

$\Rightarrow \underline{x}^k < \frac{1}{2} < \bar{x}^k$ is s.t. $x_i < \underline{x}^k \Rightarrow$ I eliminated for i

$x_i > \bar{x}^k \Rightarrow$ NI eliminated.